MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXIV. JUNE 12, 1914. No. 8

Major E. H. HILLS, C.M.G., D.Sc., F.R.S., PRESIDENT, in the Chair.

Jan van der Bilt, Observatory, Utrecht, Holland; and
Frank Basil Cooke, Observatory, Sydney, New South Wales, Australia,

were balloted for and duly elected Fellows of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended:—

Dr. Eugène de Krudy, Director of the Flammarion Observatory, Bâle, Switzerland (proposed by Camille Flammarion).

Seventy presents were announced as having been received since the last Meeting, including, amongst others:—

A. C. D. Crommelin, The Star World, presented by the author; Oxford University Observatory, Miscellaneous Papers, Vol. IV., presented by Prof. Turner; Astrographic Chart; forty-one charts, presented by the Royal Observatory of Belgium; Helwân Observatory, Photographs of Nebulæ taken by Mr. Knox Shaw; seven lantern slides, presented by the Observatory.

Correction to "Note on Spectroscopic Binaries and the Velocity of Light." By R. S. Capon.

(*Communicated by Professor H. H. Turner.*)

Professor H. C. Plummer has kindly called my attention to an error in the deduction of the Doppler effect on the Ritz hypothesis in the previous note. The Doppler effect deduced there (*M.N.*, 1914 April, pp. 507 and 508) applies only to the immediate neighbourhood of the source. It may readily be seen, however, that when the observer is at a great distance from the moving source, the effect is profoundly modified. For consider the source moving away from the observer with a diminishing velocity, and consider two consecutive light waves whose crests are separated by a distance d immediately after emission. The velocity of the second towards the observer will be greater than that of the first; for each is affected with the decreasing velocity of the source. The distance d will therefore steadily decrease, while the mean velocity of the pair of waves remains constant, so that the observed frequency depends on the distance of the observer from the source.

If $z+\mathrm{D}$ is the distance of the source from the observer, an impulse emitted at the time t is observed at the time

$$t' = t + \frac{\mathrm{D}+z}{c-p\dot{z}} \quad . \quad . \quad . \quad . \quad (1)$$

where c is the rate of increase of the radius of a spherical light wave, and p is a fraction, or unity for the simple Ritz hypothesis.

Differentiating this for t

$$\frac{dt'}{dt} = 1 + \frac{\dot{z}(c-p\dot{z}) + p\ddot{z}(\mathrm{D}+z)}{(c-p\dot{z})^2} \quad . \quad . \quad . \quad (2)$$

If dt is the periodic time of a light disturbance originating in the source, dt' is the observed periodic time at a distance $\mathrm{D}+z$ from the source. If y is the radial velocity of the source deduced in the usual way from the displacement of a spectral line, a well-known formula gives

$$\frac{dt'}{dt} = 1 + \frac{y}{c}.$$

Neglecting powers of $\frac{\dot{z}}{c}$ above the first, and neglecting z compared with D, we get for values of y small compared with c

$$y = \dot{z} + \frac{p\mathrm{D}\ddot{z}}{c}.$$

The assumption of the Ritz hypothesis leads therefore to an alteration of the ordinates as well as the abscissæ of the time

velocity curve of a spectroscopic binary. The velocity is not only announced at a time different from the time for the fixed ether hypothesis, but it is in addition the wrong velocity if the distance D is great, the distortion depending on the radial acceleration of the source. The existence of this second term was overlooked in the previous note.

As the right-hand side of (2) tends to zero, the observed periodic time dt' becomes zero, and therefore the deduced radial velocity tends to become the velocity of light. If $\dot{z}$ and z are not large, the apparent velocity will be the velocity of light when $p\mathrm{D}\ddot{z} = -c^2$. Considering the simple case of uniform motion in a circle of radius a, the periodic time being P days, and taking the line of sight in the plane of the circle, the apparent velocity is found to be the velocity of light if, (D) being the distance in light years,

$$(\mathrm{D}) > 10^3 \sqrt{\frac{2}{p}} \frac{\mathrm{P}}{2\pi \times 365}.$$

In making this calculation it is assumed that a subtends an angle less than $0''{\cdot}1$ at the earth, so that $\frac{\mathrm{D}}{a} > 2 \times 10^6$. P is of the order 1 day for many binaries, while (D) is of the order 300 light years. p must therefore be less than about 4×10^{-6} if very large velocities are to be avoided, a very much smaller value than that obtained in my note or in any of the papers referred to there. No doubt the improbable results following from the Doppler effect as corrected by Professor Plummer will be considered by many to be a strong argument against the Ritz theory. For it is not difficult to see that if D is great enough, and the radial acceleration of the source is variable as in circular or elliptic motion, the wave train will become folded upon itself, giving rise to very complex variations in the intensity of the light, as well as the large velocities noticed above.

This oversight only affects the disproof of the Ritz hypothesis. The evidence against the hypothesis is much more decisive than was supposed. The remaining conclusions are unaffected. For definiteness, it may be convenient to restate the conclusions here.

Conclusions.

The Ritz theory is definitely disproved. The coefficient p, for which Ritz requires the value unity, must be less than 4×10^{-6}.

The equations (2), (3), and (4) of the note offer a formal explanation of the anomaly of the periastra. Their physical interpretation is consistent with an influence of the velocity of the source on the velocity of light such as that suggested by the hypothesis of Stokes.

University Observatory, Oxford:
1904 *June* 9.

Note on the Velocity of Light and Doppler's Principle.
By H. C. Plummer, M.A., Royal Astronomer of Ireland.

1. The paper by Mr. Capon on "Spectroscopic Binaries and the Velocity of Light" (p. 507) deals with a fundamental question and is of great interest. It is, however, possible to approach the matter from a rather different point of view, and in one respect to modify the theory by which he deduces and interprets Doppler's principle on the hypothesis of Ritz.

Spectroscopic binaries have become so normal a part of the astronomical system, that it requires a mental effort to feel any suspicion of their objective reality. And yet it is well to bear in mind the basis on which the generalisation as a whole rests. The starting-point is a certain theory of light. From this follows Doppler's principle as commonly understood. Later, certain periodic changes are discovered in the spectra of stars, and these are interpreted in the light of Doppler's principle. As a consequence, the existence of spectroscopic binaries is inferred. Finally, the inference has been so widened by accumulating observations that we now believe that one quarter of all the stars are binaries. This is not merely a logical process, but the historical order of events.

If now a new theory of light is proposed, entailing a modification of Doppler's principle, it clearly cannot be tested by its success or failure in explaining the inferences from the older theory unless these inferences are independently substantiated. In point of fact, there are quite serious difficulties in accepting the binary character of certain individual stars and of certain whole classes of stars as derived from the ordinary Doppler's principle. It is unnecessary to specify these objections here. It is enough in logic that a new theory must eventually be tested by ascertained facts, and need not be dismissed simply because it is inconsistent with certain deductions from another hypothesis.

2. For simplicity, we imagine the observer at rest at O. A light impulse is started from the source P at the time t and reaches O at the time τ. The point P is moving at t with the velocity V in a direction making the angle ψ with OP. We suppose that the centre of the wave moves in the same direction with the velocity $p\mathrm{V}$, and at the time τ it reaches the position C. Hence, if c is the velocity of light and $\mathrm{OP}=\mathrm{R}$, since $\mathrm{PC}=p\mathrm{V}(\tau-t)$, $\mathrm{OC}=c(\tau-t)$, and $\mathrm{OPC}=\pi-\psi$,

$$c^2(\tau-t)^2=p^2\mathrm{V}^2(\tau-t)^2+\mathrm{R}^2+2p\mathrm{RV}(\tau-t)\cos\psi$$

or

$$(c^2-p^2\mathrm{V}^2)(\tau-t)^2-2p\mathrm{RV}(\tau-t)\cos\psi-\mathrm{R}^2=0,$$

whence

$$(c^2-p^2\mathrm{V}^2)(\tau-t)=p\mathrm{RV}\cos\psi+\mathrm{R}\sqrt{(c^2-p^2\mathrm{V}^2\sin^2\psi)}.$$

If we retain here only the terms which are of chief importance, we have simply

$$\tau = t + \frac{R}{c} + p\frac{RV}{c^2}\cos\psi \quad . \quad . \quad . \quad (1),$$

which corresponds to Mr. Capon's equation (1). It follows that

$$\frac{d\tau}{dt} = 1 + \frac{\dot{R}}{c}\left(1 + \frac{pV}{c}\cos\psi\right) + \frac{pR\ddot{R}}{c^2},$$

since $V\cos\psi = \dot{R}$, or, neglecting a relatively unimportant term,

$$\frac{d\tau}{dt} = 1 + \frac{\dot{R}}{c} + \frac{pR\ddot{R}}{c^2}.$$

Now this is the ratio of the rate at which light impulses are received at O to the rate at which they are transmitted from the source at P. Hence, if v and v_0 are the relative frequencies,

$$\frac{v_0}{v} = 1 + \frac{U}{c} = 1 + \frac{R}{c} + \frac{pR\ddot{R}}{c^2},$$

where U is the radial velocity which would be inferred by the ordinary Doppler principle ($p = 0$). Thus

$$U = \dot{R} + pR\ddot{R}/c \quad . \quad . \quad . \quad (2),$$

and we see that the Doppler principle is unaffected by the new theory in so far as a source of light in uniform motion is concerned, but that an additional term is introduced which depends on the radial acceleration. Since R/c is of the order of thirty years for the majority even of bright stars, it is obvious that this term must exercise a predominant effect if the theory of Ritz ($p = 1$) is true. At moderate distances, however, the effect can still be neglected.

3. We now consider a simple example. Let

$$R = R_0 + a\sin\frac{2\pi t}{T}, \quad \dot{R} = \frac{2\pi a}{T}\cos\frac{2\pi t}{T}, \quad \ddot{R} = \frac{-4\pi^2 a}{T^2}\sin\frac{2\pi t}{T}.$$

This is the case of a simple harmonic vibration with the period T and amplitude a. It applies either to a linear vibration of the source in the line of sight or to a source moving uniformly in a circular orbit. In the latter case, if the orbit is inclined to the line of sight, a is the projection on this line of the radius in the

position corresponding to $t = \frac{1}{4}$ T, and t is reckoned from the receding node. Then (1) and (2) become

$$\tau = t + \frac{R_0}{c} + \frac{a}{c}\sin\frac{2\pi t}{T} + \frac{2\pi p a R}{c^2 T}\cos\frac{2\pi t}{T},$$

$$U = \frac{2\pi a}{T}\left(\cos\frac{2\pi t}{T} - \frac{2\pi p R}{cT}\sin\frac{2\pi t}{T}\right),$$

where R may be considered to have its mean value. Hence, if

$$\tan\alpha = 2\pi p R/cT$$

$$\tau = t + R_0/c + a\sin(2\pi t/T + \alpha)/c\cos\alpha$$

$$U = 2\pi a\cos(2\pi t/T + \alpha)/T\cos\alpha;$$

or again, if $U_0 = 2\pi a/T\cos\alpha$,

$$\tau = t + R_0/c + U_0 T\sin(2\pi t/T + \alpha)/2\pi c$$

$$U = U_0\cos(2\pi t/T + \alpha).$$

4. Now an observed velocity curve is a (τ, U) curve. Hence U_0 is the observed amplitude of this curve. If we take U_0 to be of the order 100 km./sec., $U_0/2\pi c$ is of the order 1/20000, and this fraction of the period is absolutely negligible in the abscissæ of a velocity curve. Hence in direct opposition to Mr. Capon and some other writers, we conclude that the time scale is practically unaffected when we adopt the theory of Ritz. But on the other hand, and again in opposition to these writers, we find that the Doppler principle is profoundly modified. For the observed amplitude is not $V_0 = 2\pi a/T$ as it would be interpreted for a spectroscopic binary in the light of the ordinary theory, but $V_0 \sec\alpha$. Here α is an angle in the neighbourhood of $\frac{1}{2}\pi$ on the theory of Ritz. If $p = 1$, $R/c = 30$ years, and $T = 30$ days, $\tan\alpha$ is of the order 2000. Hence the real vibration of the source is magnified by a factor of this order when stellar spectra are interpreted in the ordinary way. An apparent amplitude of 10,000,000 km. may thus have its origin in a real amplitude of 5000 km. or less. It follows that if the new theory came to be established on independent evidence, we should be compelled to contemplate the necessity of giving up the hypothesis of binary systems as a common type among the stars. On the contrary, it would be more plausible to attribute the vibration to a mere radial pulsation in the atmosphere of the star. For particular classes of variables this alternative theory would appear entirely suitable, and there is nothing against it in the case of an ordinary star which shows only one spectrum. The anomaly referred to by Mr. Capon ceases to be a crucial test, and no fictitious eccentricity arises from a circular orbit.

5. What tests may be substituted will be considered immediately. But it may be of interest to consider the order of the effect to be expected in the case of the Sun. With this object we put $p = 1$, $T = 25$ days, $a = 700{,}000$ km., and $R/c = 500$ seconds. Then

$$\tan\alpha = \frac{1000\,\pi}{25 \times 86400} = \cdot00145,$$

or $\alpha = 5'$, sec $\alpha = 1{,}000{,}001$. The apparent effect is therefore exaggerated by one part in a million and the maximum phase altered by 5′ (in heliographic longitude). It seems clear that so small an effect must escape detection. The effect in question must be tested at stellar distances.

6. To represent the case of the nearest visual binary, α Centauri, we may take $R/c = 4$ yrs. and $T = 80$ yrs. Hence, if $p = 1$, tan $\alpha = \pi/10$, $\alpha = 17^{\circ}{\cdot}5$, sec $\alpha = 1{\cdot}05$. Or again, for Sirius, $R/c = 7{\cdot}5$ yrs., $T = 50$ yrs., tan $\alpha = 3\pi/10$, $\alpha = 43^{\circ}{\cdot}3$, sec $\alpha = 1{\cdot}37$. Both these orbits have considerable eccentricities, and the simple theory only suggests the order of the effects to be expected. Wright's parallax for α Centauri, in excellent agreement with the heliometer result, may not in itself dispose of the suggested theory, and Campbell's comparison of the observed radial velocities of Sirius with those calculated from the elements of the visual system may cover too short an interval of time to be decisive. But it seems clear that continued spectrographic observations of a suitable visual binary would be quite capable of deciding the question (chiefly on the evidence of the phase of the motion) without introducing any extraneous hypothesis whatever.

7. When we return, however, to the consideration of the spectroscopic binary, we see at once the grave difficulties involved in accepting the theory of Ritz. At the outset, the case for approaching the question with the utmost freedom from preconceptions was stated emphatically. But with the modification which has been introduced into the interpretation of the displacements of spectral lines, the absolutely revolutionary character of the hypothesis is immediately evident. We might welcome an optional alternative in the explanation of the observed displacements. But there is no option. The theory requires that the periodic motion of the source shall be magnified by a large factor varying as the distance of the star and inversely as the period. The original motions must therefore be exceedingly small. We are then left at a loss to account for all those stars which show double spectra. They cannot all be binary systems whose planes are nearly perpendicular to the line of sight. In particular, the eclipse effect verified by Stebbins in the case of β Aurigæ at once confirms the binary character of the system and disproves the possibility of a high inclination for the orbit. Moreover, in this instance and in the case of all eclipse variables the phase of the orbit observed spectrographically differs by a quarter of a period from what has been supposed and comes into direct opposition to the alignment of the eclipsing body. Hence, unless we are prepared to deny the existence of close binary systems altogether and abandon equally the theory of eclipsing variables, we come, by a slightly different path, to the same conclusion as Mr. Capon, that the theory of Ritz is absolutely untenable.

Dunsink Observatory.

Note on the Number of Components of a Compound Periodic Function. By J. B. Dale, M.A.

In the paper on the "Resolution of a Compound Periodic Function" (*Mon. Not.*, vol. lxxiv. p. 636), a criterion for the number of components was established in the form of an inequality satisfied by the ratio of two determinants. It was there shown that if the tabulated values of the function to be analysed are assumed to be correct to the nearest unit, we may at the outset take the number of distinct periodic terms to be n, if the ratio Δ_{n+1}/Δ'_n is less than 4^n in numerical value.

The limit 4^n is an extreme one, and could only be attained by Δ_{n+1}/Δ'_n if one of the component periodic terms had an amplitude of nearly half a unit and a period twice that of the period of analysis. In general, we find in this way a lower limit for the number of distinct periods.

We may obtain an *à posteriori* justification of the assumption made regarding the number of periods by consideration of the probable errors.

With the notation used in §§ 5, 6 of the previous paper, let ϵ be the probable error of any y. Then, if s y's are combined to form y', the probable error of y' is $s^{\frac{1}{2}}\epsilon$, and if a is the r^{th} difference of y', the probable error of a is $s^{\frac{1}{2}}2^{\frac{r}{2}}\epsilon = \epsilon_1$. The probable error of $\mathrm{E}^n a$ is $2^{\frac{n}{2}}\epsilon_1$.

If there are n periodic terms, then the probable error in the value of

$$\phi(a_r) \equiv \mathrm{E}^n a_r + p'_1\mathrm{E}^{n-1}a_r + p'_2\mathrm{E}^{n-2}a_r + \ldots + p'_{n-1}\mathrm{E}a_r + p'_n a_r$$

is $$(2^n + p'^2_1 2^{n-1} + p'^2_2 2^{n-2} + \ldots + p'^2_{n-1}2 + p'^2_n)^{\frac{1}{2}}\epsilon_1 = \epsilon_2.$$

Hence the assumption of n periods is justified if for all the values of r, $\phi(a_r)$ is less than or does not much exceed ϵ_2.

The application of this test to the example discussed in § 9 of the paper referred to, shows that an assumption of two periods is reasonable if the probable error of Newcomb's residuals is greater than 0″·26.

1914 *June* 9.

The Fluctuations in the Epochs of Sun-spot Maxima and Minima. By J. B. Dale.

1. The present paper gives the results of an examination of the dates of the sun-spot maxima and minima, undertaken with a view to ascertaining whether their deviations from a uniform arithmetical progression could be represented by a periodic formula such as that employed by Chandler for representing the epochs of the maxima of variable stars. The problem has already been attacked by Professor Turner (*Mon. Not.*, lxviii. p. 656), who used the

periodogram. The method of analysis employed in the present investigation is that developed in a recent paper (*Mon. Not.*, lxxiv. p. 628), and it will be found that while the results obtained agree in one respect with those arrived at by Professor Turner, on the whole there is considerable divergence. The data used are those given by Dr. Wolfer and reprinted in Table I. of Professor Turner's paper, with the addition of +1·4 at the end of the list of minima to represent the 1900 epoch. In addition, a second investigation of the period from 1750 to 1894 was made, in which the epochs of maxima and minima were deduced from the mean frequencies of the years preceding, including and following the turning value.

2. If we assume that the sun-spot frequency in the neighbourhood of a maximum or minimum can be expressed in the form

$$y = a + bt + ct^2,$$

and if s_{-1}, s_0, s_1 represent the mean frequencies for the three years, of which that including the minimum or maximum is the central one, then

$$s_{-1} = a - b + \frac{13}{12}c,$$

$$s_0 = a + \frac{1}{12}c,$$

$$s_1 = a + b + \frac{13}{12}c,$$

and the turning value occurs at a distance

$$\frac{s_{-1} - s_1}{2(s_{-1} + s_1 - 2s_0)}$$

from the middle of the central year. With the above assumption, the dates of maxima and minima are those shown in the following table :—

Table I.

Max.	Min.	Max.	Min.
1750·1	1755·8	1830·1	1833·8
1761·5	1766·3	1837·3	1843·8
1769·9	1775·6	1848·5	1856·1
1778·7	1784·5	1860·1	1867·2
1788·0	1798·5	1870·7	1878·8
1804·5	1810·6	1884·0	1889·4
1816·7	1823·2	1893·6	1901·7

These dates differ but little from those assigned by Wolfer, and the resulting periods are in close agreement, except in one instance where Wolfer's data lead to an imaginary period, while the above give a real one.

3. The analysis proceeds on the assumption that the epochs can be expressed by a formula

$$T = A_0 + A_1 n + \sum_{r=1}^{r=4} B_r \sin(\theta_r n + \gamma_r),$$

where A_1 is the main period in years, n the number of periods elapsed since the epoch A_0, and B_r, θ_r, γ_r constants. Assuming four periodic terms, fourteen constants have to be determined, and inserting in the above equation the successive values of T with the corresponding values of n, we form fourteen equations from which the constants can be obtained.

4. The main points in the analysis of the maxima are set out below, the notation being that employed in the paper already cited.

TABLE II.

n	y	Δy	$\Delta^2 y = a$	Ea	E^2a	E^3a	E^4a
0	17501	114	−30				
1	17615	84	+ 4	− 25			
2	17699	88	+ 5	+ 76	− 63		
3	17787	93	+72	− 38	+160	−206	
4	17880	165	−43	+ 84	−143	+296	−512
5	18045	122	+12	−105	+136	−306	+514
6	18167	134	−62	+ 52	−163	+218	−496
7	18301	72	+40	− 58	+ 82	−190	+283
8	18373	112	+ 4	+ 30	− 27	+ 65	
9	18485	116	−10	+ 31	− 17		
10	18601	106	+27	− 47			
11	18707	133	−37				
12	18840	96					
13	18936						

The quartic which determines the periods is

$$\begin{vmatrix} z^4 & z^3 & z^2 & z & 1 \\ +512 & -296 & +143 & -84 & +43 \\ +514 & -306 & +136 & -105 & +12 \\ +496 & -218 & +163 & -52 & +62 \\ +283 & -190 & +82 & -58 & +40 \end{vmatrix} = 0$$

and this reduces to

$$z^4 + 1{\cdot}1385z^3 - 2{\cdot}467z^2 - 1{\cdot}4727z + 1{\cdot}2522 = 0.$$

The roots are

$$z_1 = 1{\cdot}2059, \quad z_2 = 0{\cdot}5408, \quad z_3 = -1{\cdot}042, \quad z_4 = -1{\cdot}843.$$

These are respectively twice the cosines of the corresponding angles θ, and the values of these angles are

$$\theta_1 = 52^\circ\cdot9, \quad \theta_2 = 74^\circ\cdot3, \quad \theta_3 = 121^\circ\cdot4, \quad \theta_4 = 157^\circ\cdot1.$$

The constants B_1, γ_1 are obtained by means of the formulæ

$$B_1 = \frac{(Q_1^2 + P_1^2 - 2P_1Q_1\cos\theta_1)^{\frac{1}{2}}}{2^5 \sin\theta_1 \cdot \sin^2\frac{\theta_1}{2} \prod_{r=2}^{r=4} (\cos\theta_1 - \cos\theta_r)},$$

$$\tan\gamma_1 = -\frac{Q_1 \sin 5\theta_1 - P_1 \sin 3\theta_1}{Q_1 \cos 5\theta_1 - P_1 \cos 3\theta_1},$$

with similar expressions for B_2, B_3, B_4, γ_2, γ_3, γ_4. When all these quantities have been determined, A_0 and A_1 can be obtained immediately.

The resulting expression for the epochs of maxima is

$$\begin{aligned} T_M = {} & 1746\cdot8 + 11\cdot38n \quad . \quad . \quad . \quad . \quad . \quad . \quad (1) \\ & + 2\cdot3 \sin(52^\circ\cdot9n + 70^\circ\cdot5) + 1\cdot6 \sin(74^\circ\cdot3n + 12^\circ\cdot7) \\ & + 0\cdot2 \sin(121^\circ\cdot4n + 33^\circ\cdot8) + 1\cdot2 \sin(157^\circ\cdot1n + 36^\circ\cdot6). \end{aligned}$$

The epochs of minima given in Table I. treated in the same manner yield

$$\begin{aligned} T_m = {} & 1753\cdot3 + 11\cdot40n \quad . \quad . \quad . \quad . \quad . \quad . \quad (2) \\ & + 1\cdot7 \sin(56^\circ\cdot7n + 41^\circ\cdot4) + 1\cdot7 \sin(68^\circ\cdot2n + 109^\circ\cdot6) \\ & + 0\cdot5 \sin(134^\circ\cdot7n + 213^\circ\cdot3) + 0\cdot5 \sin(156^\circ\cdot3n + 174^\circ\cdot1). \end{aligned}$$

5. Before discussing these formulæ, it is desirable to state the results of the investigation of Wolfer's epochs. For the purpose of analysis each set was divided into two parts containing fourteen terms covering a period of about 150 years. One term was common to the two groups of the minima, and two terms to the two groups of the maxima.

Equations were formed as above for the arguments of the periodic terms, and the results are given in Table III. It became clear that the data, especially for the period 1600–1750, were so much affected by accidental errors that it was not worth while completing the solution for the other constants.

TABLE III.

Maxima.			Minima.		
I.	II.		I.	II.	
	A.	B.		A.	B.
32°·3	51°·6	52°·9	54°·6	55°·2	56°·7
85°·8	70°·7	74°·3	103°·2	72°·5	68°·2
124°·7	Imag.	121°·4	(120°·2)	118°·7	134°·7
135°·4	156°·2	157°·1	(132°·1)	148°·5	156°·3

The columns headed I. refer to the earlier period of 150 years, and those headed II. to the later period. Columns I. and II. A are derived from the Wolfer data already analysed by Professor Turner, and columns II. B from the data in Table I.

6. Formulæ (1) and (2), § 4, agree in making the value of the 11-year period almost exactly 11·4. This accords with the value found in the recent investigations by Schuster, Turner, and A. A. Michelson; but it is very difficult to see how it can be made to fit the earlier observations. Indeed the formulæ fail to represent the epochs during the first century and a half, and if the 11-year period is constant it must be nearer 11·13 than 11·40. Is the period slowly lengthening? Chandler found it necessary to make this assumption in the case of some variables.

7. An inspection of the arguments of the periodic terms shows such good accordance between the maxima values and the minima ones, that we may reasonably assume that the differences are accidental, and due to the imperfect character of the data. Columns I. are not altogether satisfactory, but this is only to be expected. The accidental errors of the earlier observations are doubtless considerable.

8. The first term to be considered has an argument between 52° and 55°, with a preponderance of evidence in favour of the larger value. The length of the corresponding fluctation may have any of the values $11{\cdot}4 \times 360^\circ/(m360^\circ \pm 55^\circ)$ where m has positive integral values. If we take $m=0$, we obtain 74·5 years. The smaller value 52° gives 78·5 years.

From formulæ (1) and (2) we find the corresponding amplitudes to be 1·7 and 2·3 years. A value 2·0 years would apparently satisfy both within the limits of accidental errors.

This term is found by Professor Turner, his value for the argument being 53° and for the amplitude about one year.

His other term, with argument 20° and amplitude also about one year, I have failed to detect.

9. The next term to be considered is that with argument about 72°. This appears in both the maxima and minima of the last century and a half. It may be represented by the angle 85°·8 in the earlier maxima, but does not appear in the earlier minima (unless 103°·2 is the corresponding angle greatly displaced by accidental errors). Formulæ (1) and (2) accord excellently in assigning an amplitude of 1·6 years to this term. The period of this fluctuation may be 57 years, but, as explained above, it may be less.

This term was not obtained by Professor Turner, but he points out that an extension of the theoretical formula for variable stars to the case of the Sun would give a periodic term with argument 76°.

10. There are traces of a term with an argument about 122°. The amplitude assigned by formula (1) is only 0·2 years, and by formula (2) 0·5 years. Accidental errors would therefore greatly affect it. It appears in Max. I. and II. B. The corresponding

root in the equation for II. A gave an imaginary angle $\cos^{-1}(-14{\cdot}8)$. In the solution for Min. I., the term appeared with an exponential factor in the form $(1{\cdot}46)^n \sin(120^\circ{\cdot}2n+\gamma)$, but an inspection of the equation showed that comparatively small changes in the coefficients would yield a solution free from exponentials. On the whole, the evidence for the existence of this term is good. The length of the corresponding fluctuation would be about 33·5 years.

We notice, however, that in both the solutions for the maxima and in one of the solutions for the minima, an angle whose magnitude is about 134° appears. In Min. II. B it replaces the angle 119° which appears in II. A. In Min. I. it is associated with an exponential factor $(1{\cdot}05)^n$. It is possible that this may represent another periodic term whose amplitude is nearly the same as that of the 122° term.

11. There also seems to be considerable evidence for the existence of a term with argument 156°, but formulæ (1) and (2) disagree in respect of the corresponding amplitude. Taking the mean, its value would be about 0·9 years, and the period of this fluctuation would be about 26 years.

12. The difference of the epochs in formulæ (1) and (2) is 6·5 years, indicating that while on the average this is the time which elapses between a maximum and the succeeding minimum, the time from minimum to maximum is only 4·9 years. One would expect the phases of the terms in the minimum formulæ to be the fraction 6·5/11·4 of their arguments in advance of the corresponding maximum phases. This is not the case, and I have been unable to discover any relation between the two groups of phases.

13. The question arises whether any relation exists between the arguments of the periodic terms. Now $\frac{1}{3}\times 360^\circ = 120^\circ$, $\frac{1}{5}\times 360^\circ = 72^\circ$, and $\frac{1}{7}\times 360^\circ = 51^\circ{\cdot}4$, and arguments have been found very near to these values. The coincidence may be purely a chance one, but it suggests that it is desirable to investigate the matter further, in particular to see whether corresponding fluctuations can be detected in the earth's magnetism.

It may also be noted that the period of 33·5 years corresponding to the argument 122° is close to the usually accepted value of the length of Brückner's cycle.

14. On the whole, the evidence of the existence of periodic terms appears to be fairly strong, but the data seem to be affected with accidental errors of such amount that the amplitudes and phases cannot be determined with sufficient accuracy to enable one to construct a formula which could with any degree of confidence be used for purposes of prediction. Formula (1) would make the date of the last maximum 1908·3, which is certainly late. Formula (2) gives 1913·5 as the date of the most recent minimum, and this appears to agree quite well with the observations.

University of London,
King's College:
1914 *June* 9.

An Area of Long-continued Solar Disturbance, and the Associated Magnetic Storms. By the Rev. A. L. Cortie, S.J. (Plates 14, 15.)

1. The most important outburst of sun-spots in the year 1910 was that represented by the appearance of the groups (Greenwich numbers) 6893 and 6894. These two groups, the latter of which was the largest spot-group observed during the year, formed together a fine curved stream of spots, which covered a belt on the Sun's surface 17° in longitude and 7° in latitude. The former of these two groups was first seen on the east limb on September 26, and last seen on the west limb on October 7; the latter was observed from September 27 to October 9. Group 6894 was, moreover, the third successive appearance of a group whose periods of transit of the Sun's visible hemisphere as group 6874 was from August 3 to August 14, and again as group 6880 from August 31 to September 11. Its companion and preceding group 6893 reappeared as a small spot 6903, seen only on October 24. In the next rotation of the Sun some groups of small spots, 6914 and 6915, marked the seat of the disturbance, while in yet another rotation another group of small spots, 6919, seen December 18 to December 22, was within the limits of the area covered by the great outburst of October. On October 1 the limits of the measured positions of the components of group 6893, given in the Greenwich results, were from longitude 60°·3 to 54°·2, and from latitude −13°·0 to −11°·0; and of those of group 6894 from longitude 51°·7 to 43°·0, and from latitude −18°·3 to −12°·9.

It not unfrequently happens, when a greater disturbance of this kind affects any area of the Sun's surface, that it is preceded by intermittent disturbances, either in the form of spots or of faculæ, for some considerable period before the appearance of the greater disturbance. Confining our attention to the limits of longitude and of latitude within which the components of the groups 6893 and 6894 were confined, it is found that this particular belt on the Sun's surface was continuously disturbed, either by faculæ and flocculi, or by spots from 1909 January 8 to 1910 December 27—that is, so far as the visible hemisphere of the Sun was concerned. All these successive, and not merely intermittent, disturbances may undoubtedly be regarded as associated together, and as indicating a specially disturbed area of the Sun's surface. These disturbances are grouped into two special regions of activity, the one connected with that occupied in October by spot-group 6893, and the other with that which was the seat of the activity of its greater companion 6894. We may conveniently designate these regions by the numbers I. and II., the connecting links between them being the before-mentioned groups, and the stream of spots 6847, 6848, 6849 of 1910 May 13 to 25. Another connecting link between these two sub-areas of the greater area of total disturbance is furnished by the groups

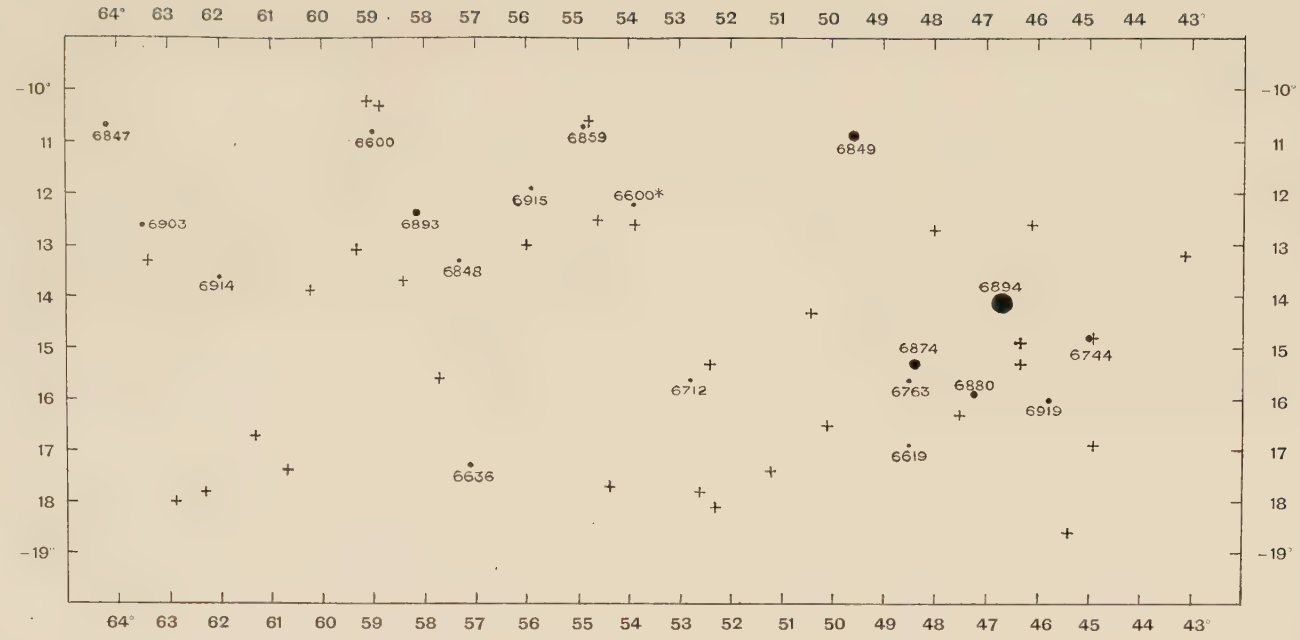
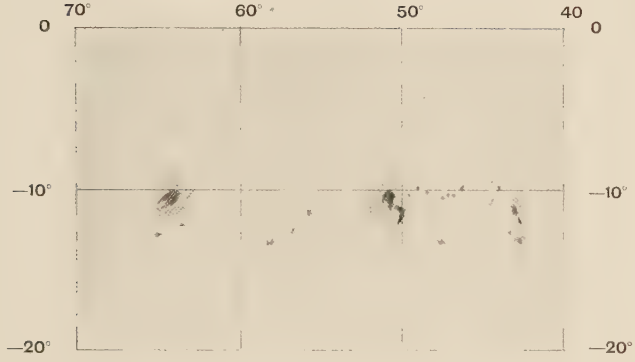
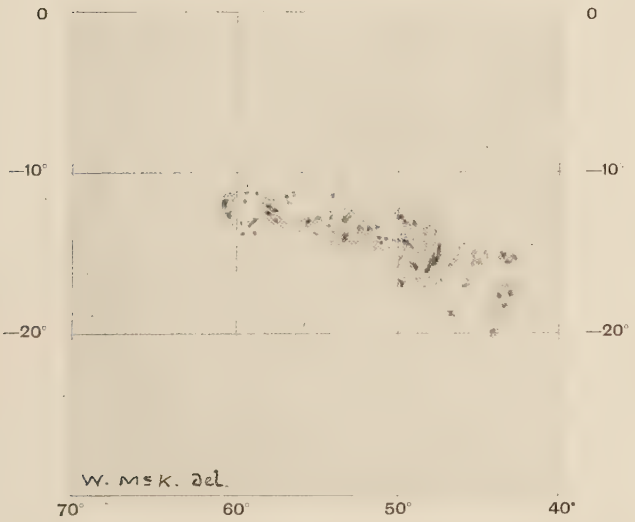


Chart of Recurrent Groups of Sun-spots and Faculæ, 1909-1910.
Longitude 64° to 43°. Latitude −10° to −19°.



1.—Groups 6847, 6848, 6849. 1910 May 16. 9-40 G.M.T.



2.—Groups 6893, 6894. 1910 Oct. 1, 9-40 G.M.T.

6849 and 6859, the latter group in region I. being set down in the Greenwich results as the return of the former, which belongs to region II. The limits of longitude and of latitude for the groups of spots and faculæ which appeared in region I. are approximately longitude 64° to 53°, and latitude − 10° to − 18°; those for region II. longitude 53° to 43°, and latitude − 13° to − 18°. The successive disturbances that marked region II. were more compact than those of region I. The following tables contain the mean positions of the several groups of spots and faculæ in the two regions, derived from the Greenwich records. With regard to the faculæ, a group recognised as identical for several days in succession, or on the east and west limbs of the Sun, has a mean value of its several separate positions set down in the tables. Several recurrent appearances of groups of faculæ are noted in the columns of remarks. These positions are also plotted on the accompanying chart (Plate 14), the larger spot-groups being represented not only in position but in relative area, in terms of the $\frac{1}{3000}$th of the visible hemisphere of the Sun. The positions of the groups of faculæ are indicated by crosses. But both in the tables and on the chart, only such groups of faculæ are registered as were independent of spots, and served to maintain the continuity of activity of the region. The spot-groups were, of course, each associated as usual with a concurrent group of faculæ in approximately the same mean position.

The spot-groups 6847 to 6849, and 6893, 6894 of May and October, which each covered the area under consideration, are also represented as drawn at Stonyhurst on 1910 May 16, and 1910 October 1.

2. *Life-History of the Disturbances.*—In 1909 January to March both the sub-regions I. and II. of the area of activity were marked by appearances in each rotation of the Sun of either faculæ or small spot-groups. The region I. continued to be disturbed until the end of July 1909, region II. until the end of March. At the end of July region I. remained dormant until the end of the year, with the exception of the appearance of a group of faculæ in November. The seat of activity was transferred to region II., which was disturbed from August to the end of December. The most important spot in this season of activity was 6744 which attained an area 1·1 in the unit $\frac{1}{3000}$th of the visible hemisphere. Table II. shows a striking series of recurrences and reappearances of spots and faculæ connected with 6744 from October to December. The mean position of this group was extremely near that of 6894 of 1910 October when the activity of the whole disturbed area attained its maximum.

The alternations of activity noted above continued in the first months of 1910, region II. becoming dormant during January when region I. was active, and region I. in its turn remaining inactive during February to April, when region II. resumed its disturbed condition. In 1910 May appeared the stream of spot-groups 6847, 6848, 6849, represented on Plate 15, which spanned

TABLE I.

Mean Positions of Groups of Sun-spots and Faculæ of Region I.
Longitude 64° to 53°, Latitude −10° to −18°.

No.	Date.	Disturbance.	Mean. Longitude.	Mean. Latitude.	Remarks.
			°	°	
1	1909 Jan. 8–17	Faculæ	59·3	−13·1	E. and W. precedes 6600.
2	9–11	6600	59·0	−10·8	
3	14	6600*	53·9	−12·2	
4	18	Faculæ	53·9	−12·6	W. connected with 6600.*
5	Feb. 14	,,	63·4	−13·3	W. return of 1, second appearance.
6	Mar. 2–12	,,	60·7	−17·4	E.W. precedes 6636.
7	4	,,	59·1	−10·2	E. return of 5, third appearance.
8	5–18	6636	57·1	−17·3	
9	Apr. 30–May 10	Faculæ	61·3	−16·7	E.W. return of 6, second appearance.
10	27	,,	58·4	−13·7	E. recurrence of 7.
11	May 25–26	,,	54·4	−17·7	E.
12	June 3	,,	62·3	−17·8	W. recurrence of 9.
13	20–22	,,	57·7	−15·6	E. return of 11, second appearance.
14	21	,,	62·9	−18·0	E. return of 12, second appearance.
15	July 27	,,	60·2	−13·9	W. region of 6893. Dormant till November.
16	Nov. 5–11	6712	52·8	−15·6	Activity in region II. until end of December.
17	1910 Jan. 25–Feb. 3	Faculæ	54·8	−10·6	Dormant until April.
18	Apr. 26	,,	58·9	−10·3	
19	May 13–19	6848	57·3	−13·3	Forms a stream with 6847, 6849. See the Plate.
20	June 9–18	6859	54·9	−10·7	Return of 6849 of region II.
21	Sept. 26–Oct. 9	6893	58·1	−12·4	Recurrence of 6848.
22	Oct. 24	6903	63·5	−12·6	Return of 6893.
23	Oct. 27–Nov. 3	Faculæ	54·6	−12·5	Region of 6893.
24	Nov. 20–24	6914	62·0	−13·6	
25	23–26	6915	55·9	−11·9	Recurrence of 6893.
26	Dec. 19–27	Faculæ	56·0	−13·0	Region of 6915.

TABLE II.

Mean Positions of Groups of Sun-spots and Faculæ of Region II.
Longitude 53° *to* 43°, *Latitude* −13° *to* −18°.

No.	Date.	Disturbance.	Mean. Longitude.	Mean. Latitude.	Remarks.
			°	°	
1	1909 Jan. 8	Faculæ	43·1	−13·2	E.
2	Feb. 11–16	6619	48·5	−16·9	
3	Mar. 6	Faculæ	44·9	−16·9	E. region of 6619.
4	31	,,	45·4	−18·6	E. return of 3. This region quiet until August. Activity transferred to region I.
5	Aug. 22–23	,,	52·3	−18·1	W.
6	Sept. 11	,,	51·2	−17·4	E. return of 5.
7	Oct. 8	,,	46·3	−15·3	E. precedes group 6744.
8	14–19	6744	45·0	−14·8	
9	Nov. 4–14	Faculæ	47·5	−16·3	E. return of 7.
10	5–11	6763	48·5	−15·6	Return of 6744.
11	12	Faculæ	52·6	−17·8	W. recurrence of 6.
12	Dec. 1–2	,,	52·4	−15·3	E. return of 11.
13	3–10	,,	46·3	−14·9	E.W. return of 9, third appearance.
14	29–30	,,	48·0	−12·7	E. return of 13, fourth appearance.
15	1910 Feb. 21	,,	50·4	−14·3	E.
16	Mar. 20–31	,,	44·9	−14·8	E.W. region of 6744.
17	Apr. 17–18	,,	50·1	−16·5	E. region of 6763.
18	May 13–25	6849	49·6	−10·9	Return as 6859, region I. Dormant until July. Activity in region I.
19	July 10–18	Faculæ	46·1	−12·6	E.W.
20	Aug. 3–15	6874	48·4	−15·3	
21	Sept. 1–11	6880	47·2	−15·9	Return of 6874.
22	Sept. 27–Oct. 9	6894	46·7	−14·1	Return of 6880, third appearance. Activity transferred to region I.
23	Dec. 18–22	6919	45·8	−16·0	Recurrence of 6880.

the whole area of activity; and from 1910 May to December, either one or other, or both regions I. and II. were marked by outbursts of spots, the culmination of activity being reached in October, when the fine stream 6893, 6894 (also represented on the plate) again spanned the active area.

This group of spots was visible to the naked eye. The appearance, therefore, of the greater disturbance was not something sudden and unexpected, but was the outcome and result of a gradual increase in activity of the area discussed for a year and nine months, especially in the sub-region II., which was the seat of the appearance of the greater spot-group of 1910 October. After October the activity gradually died out, the spot-groups of November and December being very small. The same general sequence of disturbances is noticeable also in the number and violence of the accompanying magnetic storms from May to December.

3. *The Flocculi.*—The Tortosa records of solar phenomena began with the year 1910. There are no coincident groups of flocculi in the regions discussed until group 35, March 21 to 31, which appeared in region II. with the faculæ of the same rotation. These faculæ and the flocculi marked the spot which was active as group 6744, 1909 October 14 to 19, and was again to be active as the recurrent groups 6874, 1910 August 3 to 15; 6880, 1910 September 1 to 11; and the great spot 6894, 1910 September 27 to October 9.

It is not unlikely, though not absolutely demonstrable, that the first members of the series of accompanying magnetic disturbances were synchronous with these appearances of faculæ and of flocculi. From 1910 May to December the area was continuously marked by appearances of flocculi which are as follows in the Tortosa records :—

Groups of Flocculi.	Appearance.
59	1910 May 15–20.
69	June 8–20.
81	July 7–17.
93	Aug. 4–15.
102	Aug. 31–Sept. 11.
111 112	Oct. 1–9.
121	Oct. 23–Nov. 3.
134	Nov. 21–Dec. 1.
140	Dec. 18–29.

With regard to the flocculi groups 111 and 112 which accompanied the great spot-groups 6893, 6894, they were compact round the groups during their first transit across the disc, but in the next rotation, when the small spot 6903, seen only on October 24, marked the seat of the leading group of the two, 6893, they

were diffuse and extended. With regard to these same groups of flocculi Dr. Slocum (*Astrophysical Journal*, 36, 4, pp. 265–266) remarks: "The whole disturbed area was covered with calcium flocculi extending from latitude $-10°$ to latitude $-25°$, and from longitude 32° to longitude 65°. . . . The same region returned to view on October 22, but the big spot had vanished and only a large area of bright flocculi remained to mark the place. Throughout its history the group was the scene of great activity. Bright eruptions were continually appearing and disappearing, and on several dates dark flocculi were observed in the immediate vicinity. Photographs of the H and K regions of the spectrum of various parts of the spot-group show distortions and displacements corresponding to velocities up to 40 km. per second."

4. *The Prominences.*—Several remarkable prominences were observed in the same region of activity. Dr Slocum in his paper on "The Attraction of Sun-spots for Prominences" (*loc. cit.*, p. 266) writes: "Active prominences were observed in this region from August 2 to November 5, but the best displays occurred on the west limb on October 8, and on the east limb on October 22." These prominences were evidently connected with the set of recurrent groups 6874, 6880, and 6894, and with the small group 6903, the return of 6893. "This group of prominences of October 8," continues Dr Slocum, "extended from latitude $+6°$ to latitude $-27°$ on the west limb. When the same region came to the east limb on October 22, a similar display of prominence activity was observed, extending from latitude $+12$ to latitude $-36°$. The sun-spot was no longer visible, but the prominences were gathered around the extensive area of bright flocculi, which still marked the place." This same group of prominences is illustrated in Mrs. Evershed's paper on "Some Types of Prominences associated with Sun-spots" (*Monthly Notices, R.A.S.*, lxxiii. 6, pp. 422 *et seq.*), on the dates September 26 and October 9 (figures 5, 6, 7, 18, 19). The centre of the prominences of September 26 corresponds with the centre of the spot-group 6893. The form of the prominence structure is of a set of streaks radiating from this centre, but "more nearly horizontal to the Sun's surface." With regard to the type of prominences in general Mrs. Evershed writes (*loc. cit.*, p. 422): "It is distinguished by having a definite centre, often marked by an exceedingly brilliant point or short jet rising from the chromosphere; and from this centre streaks and streamers diverge both north and south. Spectroscopic observations often indicate motion in the line of sight also, so that we may conceive the disturbance as radiating in every direction."

5. *Comparison of Results for the Solar Disturbance.*—We may regard the Sun-spot groups 6893, 6894, which represented the culmination of solar surface activity in a relatively restricted region, as marking a distinct focus of disturbance. It will be useful to compare the area occupied by the components of these groups with that covered by the accompanying flocculi, and in latitude, with the arc of prominence activity.

	Limits. Longitude.	Latitude.	Arc.
Sun-spot group	62° to 43°	−12° to −18°	6°
Calcium flocculi	65° to 32°	−10° to −25°	13°
Prominences, first appearance	...	+8° to −27°	35°
Prominences, second appearance	...	+12° to −36°	48°

The comparative results show the spreading of the disturbance outwards as higher solar levels are reached. Moreover, the prominences radiated in all directions from the spot-groups as a centre of activity.

6. *The Associated Magnetic Disturbances.*—The magnetic disturbances associated with the solar disturbances from 1910 March 20 to December 27, a period during which the area under consideration became more disturbed, are set forth in the following table. The penultimate column gives the mean projected heliographic latitude of the earth during each series of disturbances, either of spot or faculæ, and the last column the projected angular distance earth–spot.

TABLE III.

Solar Disturbance 1910 *March* 20 *to December* 27. *Associated Magnetic Storms.*

Rotation.	Groups.	1910.	Magnetic Disturbances.	Mean Heliographic Latitude of Earth.	Projected Angular Distance Earth-spot.
I.	Faculæ	March	15 16 17 18 19 20 21 22 23 24 25 26 27 m m g	−6°·9	8°
II.	Faculæ	April	17 18 19 20 21 22 23 24 25 26 27 28 29 m m m g m m	−4·9	6°
III.	6847, 6848, 6849	May	14 15 16 17 18 19 20 21 22 23 24 25 26 m m m m m g m m	−2·0	9°
IV.	6859	June	10 11 12 13 14 15 16 17 18 19 20 21 22 m g g m	+1·3	12°
V.	Faculæ	July	7 8 9 10 11 12 13 14 15 16 17 18 19 m m	+4·2	17°
VI.	6874	August	3 4 5 6 7 8 9 10 11 12 13 14 15 g m m	+6·4	23°
VII.	6880	Aug.–Sept.	30 31 1 2 3 4 5 6 7 8 9 10 11 m m	+7·2	23°
VIII.	6893, 6894	Sept.–Oct.	26 27 28 29 30 1 2 3 4 5 6 7 8 m g m m m g m g	+6·6	20°
IX.	6903	Oct.–Nov.	23 24 25 26 27 28 29 30 31 1 2 3 4 m m m g g	+4·6	18°
X.	6914	November	19 20 21 22 23 24 25 26 27 28 29 30 1 m m m	+1·6	15°
XI.	6919	December	16 17 18 19 20 21 22 23 24 25 26 27 28 m g	−1·9	14°

m = moderate, *g* = great.

The earth was most favourably situated for any directly radial action from the disturbed area during March and April when

the projected angular distance, that is the angular distance between the mean heliographic latitudes of spot or faculæ and earth, was only 8° to 6°. This angular distance increased gradually until it reached a maximum of 23° in August and September, which doubtless accounts for the paucity of the magnetic disturbances which occurred during the transit of the moderate spot-group 6880 across the Sun's disc. When the solar activity was at a maximum, September 26–October 8, this angular distance was practically unaltered, the value being 20°. But in this case the solar disturbance was very much greater, and its influence would have extended to embrace much wider limits. The comparison of the accompanying phenomena of flocculi and prominences in the last section shows this spreading out of the influence even in the Sun's immediate envelopes, and there is, in addition, distinct evidence of a divergent action from the centre of spot activity. The successive magnetic disturbances, in two groups of four days each, are in themselves further proof of such divergent action. This is entirely concurrent with what has been proved in former papers of this series (*Monthly Notices, R.A.S.*, lxxiii. Nos. 3, 6, and 7), from the study of the total solar eclipses of 1893, 1898, 1905, and 1908, from which it appeared that bundles of divergent rays in the Sun's corona took their origins from areas long disturbed by Sun-spots, which had been associated with series of magnetic disturbances. The fact that a spot centre has divergent rays as carriers of its influence, does not exclude the possibility of an emanation of strictly radial rays also, or demand that a cone of rays proceeding from it should be confined to merely terminal rays in the form of a hollow cone. In a former paper it has been shown (*Monthly Notices*, lxxiii. No. 1), that the frequency of magnetic storms is at a maximum, when the Sun-spots are congregated about the zones ± 10·0 heliographic latitude. At such periods the earth is more favourably placed relatively to the centres of spot disturbance. But even if the action be supposed to be wholly divergent, there would be many more streams at such periods than at others for the earth to encounter, especially if, as seems likely, the streams are terminated by clouds of particles. The annual variation in magnetic disturbances can be explained in this manner, when both the inclination of the Sun's axis to the ecliptic and the position of the earth relatively to the spot zones are taken into account. The results obtained in this series of papers are not, as it has been objected, unreconcilable, even on the supposition that the action from the spot centres is wholly divergent. In the present instance of the spot disturbances of 1910 September–October, had the earth been more favourably placed in heliographic latitude relatively to the spot disturbances, there is every reason to believe that the magnetic storms would have been more violent than they actually were, as the earth would have had a better chance of encountering a greater number of the streams, even if divergent, carrying the ions.

7. *Summary.*

1. A particular area of the Sun's surface was disturbed for two years, 1909, 1910.

2. A series of minor outbursts of faculæ and small spots extended over a period of seventeen months, alternating in two sub-regions.

3. The spot-groups 6893, 6894, marked the maximum phase of the disturbance.

4. The associated flocculi and prominences were extensive and widely distributed.

5. A series of magnetic storms accompanied the greater phases of the solar disturbance.

6. These indicate a divergent action from the sun-spot centres.

7. The results are consistent with those obtained in former papers of this series (*Monthly Notices*, *R.A.S.*, lxxiii. Nos. 1, 3, 6, and 7).

Stonyhurst College Observatory :
1914 *June* 5.

The Periodogram Analysis of the Variations of SS Cygni.

By David Gibb, M.A., B.Sc. (Plate 16.)

(*Communicated by Professor E. T. Whittaker, Sc.D., F.R.S.*)

§ 1. *Introductory.*

The irregular variable star SS Cygni was discovered on the Harvard photographs in 1896. Subsequent observations have been published and discussed by L. Campbell,* Parkhurst,† Parkhurst and Daniel,‡ and by the Variable Star Section of the British Astronomical Association.§

The general character of the star's variation is to remain in the neighbourhood of magnitude 11·8 for about two-thirds of the time, but at intervals of about seven or eight weeks to rise to a maximum, attaining a magnitude about 8·3. Some writers have classified these maxima as "long," "short," and "anomalous," but it is doubtful if any arrangement of the maxima into genuine natural classes can be effected, and no use will be made of this classification in what follows.

It would undoubtedly be a great advance in theoretical astronomy if some mathematical law could be found capable of expressing the variations of SS Cygni, and it was as a contribution

* *Annals Harvard Coll. Obs.*, vol. lxiv., No. ii.
† *Popular Astronomy*, vol. vi., No. iii.; vol. vii., No. iii.
‡ *Astrophysical Journal*, vol. xii. No. iv.
§ *Jour. Brit. Ast. Assoc.*, vols. xviii. to xxiv.

towards this end that the work described in the present paper was undertaken.

Assuming that a law of some sort does exist (which is a necessary presupposition to work of any kind on the subject) the light-curve of the star may be compared for a moment to the graph of the disturbance set up in the æther by an atom of, say, sodium in a flame. The graph of æthereal disturbance, if we could draw it, would not present any regular periodicity evident to a superficial inspection: but when the disturbance is analysed by the spectroscope, it is found to consist of a very large number of periodic motions superposed on each other; one such motion corresponds to each one of the "lines" in the sodium spectrum.

So long as there exists a law of any kind governing a prolonged disturbance in the æther, the disturbance when analysed by the spectroscope must give rise to a definite spectrum. The arithmetical analysis which is known as "periodogram" analysis does for curves such as those of sun-spot frequency or stellar variability exactly what the spectroscope does for æthereal vibrations; and in the present state of mathematical knowledge there is no other available method for attacking such a problem as that presented by SS Cygni. It is, of course, possible that the periodogram when it is obtained may be exceedingly complicated; but at any rate it is worth obtaining. The applicability of periodogram analysis to variable stars is strongly suggested by the fact that the simplest variable stars are strictly periodic; and anything which is a generalisation of simple periodicity should yield its secret in the periodogram.

§ 2. *The Schuster-Whittaker Process.*

The method followed in the present paper is Whittaker's modification of Schuster's original procedure. The essential point of the modification is that Whittaker does not introduce Fourier's analysis until the final stage of the investigation, whereas Schuster uses it from the beginning. As Fourier's analysis is lengthy and troublesome, Whittaker's procedure is more rapid and simple, though more tentative and less perfect in theory than Schuster's.

The process may be thus described. Let m_n denote the magnitude of the variable star on the nth day, counting from the beginning of the observations. (The star will, for the purposes of this explanation, be supposed to vary so slowly that daily intervals are sufficient.) Then in order to test whether a periodicity of, say, 50 days is one of the constituents of the star's variation, we write down the observations in rows thus:—

$$\begin{array}{cccccccccc}
m_1 & m_2 & m_3 & m_4 & \cdot & \cdot & \cdot & \cdot & m_{49} & m_{50} \\
m_{51} & m_{52} & m_{53} & m_{54} & \cdot & \cdot & \cdot & \cdot & m_{99} & m_{100} \\
m_{101} & m_{102} & m_{103} & m_{104} & \cdot & \cdot & \cdot & \cdot & m_{149} & m_{150} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}$$

Each horizontal row contains 50 consecutive observations, and is called a "lap." We take a convenient number of laps—the more the better—and then sum the numbers that stand in each vertical column of the scheme. Let the sums be denoted by

$$N_1 \quad N_2 \quad N_3 \quad N_4 \quad . \quad . \quad . \quad . \quad . \quad N_{49} \quad N_{50}.$$

Then, if there are k laps, the periodicity of 50 days will be intensified k-fold in the sequence $N_1, N_2, \ldots N_{50}$ as compared with its intensity in any one of the horizontal rows; for this periodicity enters with the same phase into every horizontal row, and its amplitude will therefore be k times as great in the sum of k rows as in a single row. On the other hand, periodicities other than the 50-day periodicity will occur with different phases in each horizontal row; and when the vertical columns are summed, the elements with different phases will, with certain exceptions to be considered separately, annul each other so that these periodicities will not appear in the sequence $N_1, N_2, \ldots N_{50}$.

Thus we have obtained a sequence $N_1, N_2, \ldots N_{50}$ in which the 50-day periodicity, if it exists, is greatly intensified, while the other periodicities are destroyed. The difference between the greatest and the least of the numbers $N_1, N_2, \ldots N_{50}$ therefore furnishes a rough indication of the amplitude of the 50-day periodicity. When the question of its existence has been settled, we can proceed to consider other periodicities. Let y denote the difference between the greatest and the least of the numbers N corresponding to the trial period x. Then, if y is plotted as a function of x, we obtain a curve which may be called a *curve of periods*. This curve will have peaks at the values of x corresponding to the periodicities which really exist: it also has smaller peaks at certain other places depending on these, but it is not necessary to enter further into a discussion of this matter here, suffice it to say that the method thus enables us to conduct a preliminary search for possible periodicities. When a number of such have been found, the more refined procedure involving Fourier's analysis is brought into operation.

§ 3. *A Mechanical Aid in performing the Process.*

The writing down of the observations in laps corresponding to the different trial periodicities is exceedingly laborious, and the present writer has devised the following means of avoiding it:—

Each observation is marked on a small wooden cube, and the cubes are arranged in rows in a wooden frame, just as in the above scheme. The advantage of using the cubes is that when a change has to be made from one trial periodicity (say 50 days) to another trial periodicity (say 51 days), the change is effected by simply sliding the cubes along in their rows, and transferring a few cubes

from the beginning of each row to the end of the row above it: no rewriting is needed.

By the aid of this device (fig. 1), and with a comptometer to

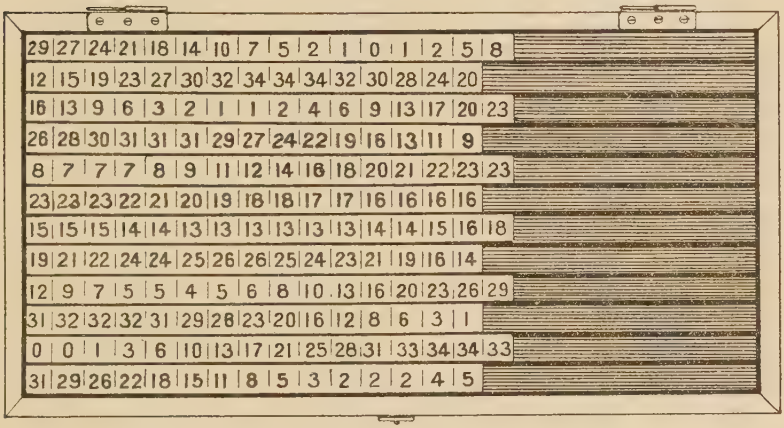


Fig. 1.

add the numbers in the vertical columns, the search for periodicities can be carried out with much greater rapidity than has been attained hitherto.

§ 4. *Application of the Method to SS Cygni.*

The first application of periodogram methods to SS Cygni was published by Whittaker * in 1911, and was based on calculations performed with 18 laps. It will appear presently that 18 laps do not give a sufficient "resolving power" to bring out the true distribution of periodicities, and the present writer has found it necessary to take as many as 98 laps. This may be compared to what has often happened in the history of spectroscopy in consequence of the increase of resolving power—*e.g.* the "*b*" group in the solar spectrum was believed in 1830 to consist of two lines; in 1840 three were recognised; in 1855 Swan found four; and so on to the complexity of the present day.

The following Table I. gives the results of a search with 98 laps. This utilises the observations from December 1896 to December 1913. The x (taken as abscissa in the diagram, fig. 2, Plate 16) is the number of days in the trial period; the y (taken as ordinate in the diagram) is the difference between the greatest and the least numbers in the sequence of sums of vertical columns obtained by adding 98 laps. The high peaks in the diagram should therefore occur at the places where marked periodicities exist.

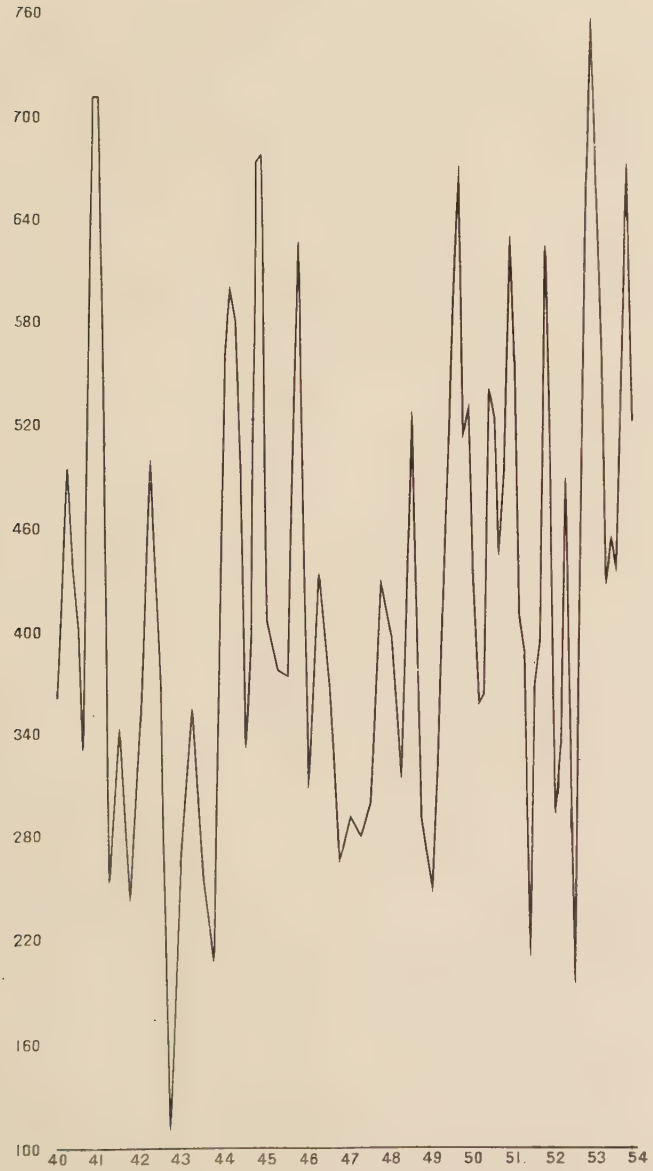
* *M.N.*, lxxi. (1911), p. 686.

TABLE I.

x.	*y*.	*x*.	*y*.	*x*.	*y*.
40	361	45	407	50½	523
¼	495	¼	378	⅝	445
⅜	436	½	374	¾	497
½	403	¾	628	⅞	630
⅝	331	46	309	51	557
¾	575	¼	434	⅛	409
⅞	711	½	366	¼	386
41	711	¾	265	⅜	210
⅛	507	47	291	½	368
¼	252	¼	280	⅝	395
½	343	½	300	¾	624
¾	242	¾	430	⅞	485
42	347	48	397	52	294
¼	499	¼	314	⅛	333
½	365	½	526	¼	487
¾	111	¾	290	⅜	320
43	264	49	248	½	194
¼	354	⅛	322	⅝	439
½	259	¼	414	¾	654
¾	207	⅜	511	⅞	757
44	557	½	586	53	657
⅛	600	⅝	673	⅛	567
¼	581	¾	515	¼	428
⅜	496	⅞	530	⅜	454
½	334	50	430	½	435
⅝	399	⅛	357	⅝	562
¾	673	¼	363	¾	672
⅞	678	⅜	539	⅞	521

It will be remarked that a notable peak is found at 40·86 days. We shall discuss in the next section whether a constituent of the star's variability really exists having this period.

A glance at the plate will show that the curve of periods is rich in "lines," and consequently requires a high "resolving power" to furnish it correctly. The resolving power of 18 laps employed by Whittaker (*loc. cit.*) was inadequate for the purpose, and this is the explanation of the discrepancy between the curve of periods published in his paper and the present curve.



The Periodogram Analysis of the Variations of SS Cygni. Fig. 2.

§ 5. *Discussion of the* 40·86*-day Periodicity.*

The following Table II. gives in column A the magnitudes of the star on 152 occasions, at intervals of 40·86 days apart; these occasions are the days indicated by the periodogram analysis as the days when the 40·86-day term has its maximum. In column B are given similarly the magnitudes of the star on 152 occasions, at intervals of 40·86 days apart, when the 40·86-day term has its minimum. The sums of column A and column B are given at the foot.

Table II.

J.D.	A.	B.	J.D.	A	B.
2413932	11·2	11·3	2415198·66	11·9	11·9
3972·86	9·4	11·3	5239·52	11·5	12·0
4013·72	11·5	10·3	5280·38	12·0	11·8
4054·58	11·5	11·7	5321·24	11·8	11·7
4095·44	11·5	10·3	5362·10	9·2	12·0
4136·30	11·6	11·8	5402·96	9·9	11·8
4177·16	11·6	11·9	5443·82	8·3	11·9
4218·02	8·5	11·7	5484·68	11·9	11·8
4258·88	11·6	11·7	5525·54	11·9	9·4
4299·74	11·9	11·9	5566·40	12·0	11·8
4340·60	9·3	11·9	5607·26	12·0	12·0
4381·46	11·9	11·1	5648·12	9·8	12·0
4422·32	11·9	11·9	5688·98	11·9	12·0
4463·18	10·2	11·9	5729·84	12·0	8·4
4504·04	11·9	11·9	5770·70	12·0	11·9
4544·90	11·9	8·4	5811·56	9·7	11·7
4585·76	11·9	11·9	5852·42	11·7	8·7
4626·62	8·5	12·1	5893·28	12·0	9·2
4667·48	12·0	8·4	5934·14	12·0	11·8
4708·34	11·9	11·9	5975·00	8·4	11·9
4749·20	11·5	12·0	6015·86	12·0	12·0
4790·06	11·9	9·6	6056·72	12·1	12·0
4830·92	12·0	11·9	6097·58	11·1	11·7
4871·78	11·5	11·7	6138·44	9·1	12·0
4912·64	8·5	11·8	6179·30	8·9	11·6
4953·50	11·9	10·8	6220·16	10·8	10·2
4994·36	11·5	9·9	6261·02	11·3	10·4
5035·22	11·9	10·0	6301·88	12·0	11·5
5076·08	11·9	11·8	6342·74	8·3	11·7
5116·94	10·0	11·8	6383·60	12·0	11·5
5157·80	8·4	11·8	6424·46	8·3	11·6

TABLE II.—*continued.*

J.D.	A.	B.	J.D.	A.	B.
2416465·32	10·5	11·8	2418058·86	10·2	11·8
6506·18	9·4	12·0	8099·72	8·6	11·6
6547·04	8·9	11·8	8140·58	11·4	11·2
6587·90	8·4	11·9	8181·44	11·1	11·2
6628·76	11·8	11·8	8222·30	8·5	10·7
6669·62	11·6	11·4	8263·16	10·7	9·7
6710·48	9·3	11·6	8304·02	10·8	10·0
6751·34	11·9	11·9	8344·88	11·1	8·9
6792·20	11·6	9·4	8385·74	11·2	8·8
6833·06	11·7	11·8	8426·60	11·2	8·4
6873·92	11·6	12·0	8467·46	11·3	11·3
6914·78	11·8	9·7	8508·32	11·3	11·2
6955·64	11·8	11·8	8549·18	9·3	11·6
6996·50	11·8	12·0	8590·04	8·6	11·7
7037·36	10·2	11·8	8630·90	11·4	11·4
7078·22	8·5	12·0	8671·76	8·5	11·4
7119·08	8·7	11·7	8712·62	11·6	11·5
7159·94	12·0	11·7	8753·48	9·8	11·6
7200·80	8·7	12·0	8794·34	8·5	11·4
7241·66	8·8	11·5	8835·20	11·3	11·8
7282·52	8·3	11·8	8876·06	11·3	11·3
7323·38	10·1	12·0	8916·92	8·9	11·8
7364·24	11·9	11·8	8957·78	11·8	11·4
7405·10	11·8	11·6	8998·64	11·4	11·8
7445·96	12·0	10·9	9039·50	8·4	11·9
7486·82	10·9	11·9	9080·36	11·8	10·3
7527·68	10·8	11·6	9121·22	11·7	11·9
7568·54	8·8	11·8	9162·08	11·8	11·4
7609·40	8·4	11·8	9202·94	11·6	11·8
7650·26	11·3	11·4	9243·80	8·4	11·8
7691·12	10·7	12·0	9284·66	11·9	9·3
7731·98	10·7	11·6	9325·52	11·6	11·9
7772·84	8·4	11·9	9366·38	10·4	11·9
7813·70	11·9	11·6	9407·24	8·5	11·8
7854·56	12·0	11·8	9448·10	11·7	11·8
7895·42	8·4	11·7	9488·96	11·8	9·4
7936·28	11·0	12·0	9529·82	11·9	11·9
7977·14	11·9	9·0	9570·68	8·6	11·9
8018·00	11·2	11·1	9611·54	12·0	11·0

TABLE II.—*continued.*

J.D.	A.	B.	J.D.	A.	B.
2419652·40	11·9	11·9	2419897·56	11·9	11·9
9693·26	12·1	9·5	9938·42	9·4	11·9
9734·12	11·9	11·9	9979·28	9·8	11·9
9774·98	9·7	11·8	2420020·14	11·8	8·6
9815·84	11·8	11·8	0061·00	11·8	11·7
9856·70	12·0	11·8	0101·86	8·8	11·8
				1634·8	1721·2

Now let x and y denote the magnitudes of the star on any two days chosen at random, and let k be a quantity such that the probability of $|x-y|$ being greater than k is equal to the probability of its being less than k. Let ξ be the sum of the magnitudes of the star on any 152 days chosen at random, and let η be the sum of the magnitudes of the star on any other 152 days chosen at random. Then, by the general theory of probability, the probability of $|\xi-\eta|$ being greater than $\sqrt{152}\,k$ is equal to the probability of its being less than $\sqrt{152}\,k$. k will in this case have a value of about 1 magnitude, so that $\sqrt{152}\,k$ may be roughly taken as 12·3. So if the days of the observations of columns A and B were selected quite at random, the "probable" value of the difference between the sums of columns A and B would be 12·3. We have, however, found that the difference of the sums of columns A and B was 86·4; which is seven times the expected quantity 12·3. Now in the general theory of errors, the probability of an error seven times the probable error is about 2×10^{-6}, and we may take this as a rough measure of the probability of the actual result on the assumption that there is no *systematic* dependence of the magnitude on a period of 40·86 days; in other words, the chances are roughly half a million to one that a term of period 40·86 days really exists in the variability of the star.

§ 6. *Remarks on the Periodograms with Different Resolving Powers.*

In the course of the work, periodograms were calculated in which the number of laps was some number other than 98; and it is interesting to compare the general form of the curves obtained in these different cases. It is found that the curves do not agree very well, even when the differences due to the alteration of resolving power are allowed for. A discordance of this character was found by Schuster* in his periodogram analysis of sun-spots, and the comments made by him on this matter are probably applicable in this case also. To return to the analogy of æthereal disturbances, or indeed of any material vibrating body such as considered in the theory of sound, we have there become

* *Proceedings Royal Society*, vol. lxxvii., A (1906).

accustomed to the idea of systems which have definite non-commensurable *periods* of vibration, but in which the *phases* and *amplitudes* of the different constituent terms are slowly or suddenly changed in the course of a long spell of vibration; these changes are due to the action of the forces which supply energy to replace that lost by dissipation. Probably something of the same kind exists in the case of variable stars and sun-spot activity. If this is so, it may be quite possible to determine the constituent *periods* by means of periodogram analysis, but the changes of phase and amplitude may make it impossible to obtain a complete permanent mathematical expression for the light curve, and so may make it impossible to construct an ephemeris.

The Mathematical Department,
Edinburgh University:
1914 *May* 1.

Three Variable Stars in the Region of χ *Persei.* (Plate 17.)

(*Communicated by the Royal Astronomer of Ireland.*)

1. The following paper contains the results of an investigation of the photographic magnitudes of certain stars in the region of the χ Persei cluster, and relates chiefly to the following three variables:—

I. B.D. $+56°{\cdot}551$ = Harvard Second Catalogue 021356. Discovered by Bailey in 1898.

II. RS Persei = B.D. $+56°{\cdot}583$ = Harvard Second Catalogue 021556 = 16·1904. Discovered by Mme. Ceraski (*A.N.* 3944).

III. SU Persei = B.D. $+55°{\cdot}597$ = Harvard Second Catalogue (suppl.) 021556b = 35·1907. Discovered by Miss Leavitt (*A.N.* 4181).

For these three stars the concluded magnitudes are given in the last three columns of Table I. In all, 176 plates of the region have been taken at Dunsink over a period of nearly four years, and they have formed the material on which a considerable amount of experimental research has been based by Professor Whittaker and Mr. Martin. All the necessary information relating to the dates and times of exposure are set out in the second, third and fourth columns of Table I.

TABLE I.

Serial No.	Date G.M.T.			Julian Day.	Exposure. min.	Magnitudes. I. B.D. +56°, 551.	II. RS Persei.	III. SU Persei.
	1909	h	m	2418				
1	May 10	11	13	437·47	60	10·14	10·64	
2	11	13	55	438·58	40	10·12	10·66	
3	July 19	13	3	507·54	60	10·10	9·94	9·80
4	22	12	12	510·51	60	10·23	9·98	9·91
5	Aug. 9	12	24	528·52	60	10·16	10·04	9·70
6	25	12	57	544·54	60	10·18	10·14	9·58
7	Sept. 7	11	13	557·47	60	10·08	10·15	9·55
8	10	10	35	560·44	60	10·09	10·22	9·5
9	13	9	56	563·41	60	10·08	10·21	9·55
10	17	10	48	567·45	60	10·09	10·21	9·56
11	20	11	48	570·49	37	10·13	10·38	9·52
12	22	10	58	572·46	60	10·15	10·27	9·47
13	Oct. 5	8	1	585·33	60	10·19	10·49	9·53
14	6	9	6	586·38	60	10·26	10·56	9·49
15	8	10	5	588·42	60	10·05	10·64	9·41
16	11	9	33	591·40	60	10·19	10·74	9·48
17	13	9	27	593·39	60	10·17	10·63	9 . .
18	18	9	27	598·39	45	10·32	10·80	9·56
19	20	9	3	600·38	60	10·28	10·52	9·41
20	21	8	48	601·37	40	10·37	10·71	
21	28	7	43	608·32	60		10·80	
22	29	7	11	609·30	70	10·47	10·73	
23	30	8	22	610·35	47	10·36	10·86	
24	31	7	56	611·33	70	10·47	10·91	9·63
25	Nov. 6	7	57	617·33	60	10·27	10·77	9·43
26	7	7	22	618·31	60	10·24	10·73	9·38
27	9	7	12	620·30	60	10·14	10·70	9·41
28	10	7	39	621·32	60	10·14	10·81	9·41
29	12	11	32	623·48	40	10·10	10·72	9·30
30	15	9	20	626·39	60	10·24	10·86	
31	16	8	14	627·34	60	10·21	10·72	9·47
32	20	7	17	631·30	60	10·23	10·63	9·47
33	23	6	26	634·27	60	10·37	10·58	
34	29	8	18	640·35	48	10·36	10·85	9·52
35	30	6	31	641·27	60	10·09	10·63	9·43
36	Dec. 1	7	11	642·30	60	10·13	10·57	9·44
37	3	11	10	644·47	60	10·13	10·66	9·51
38	4	6	52	645·29	60	10·22	10·70	9·56

TABLE I.—*continued.*

Serial No.	Date G.M.T.			Julian Day.	Exposure. min.	Magnitudes. I. B.D.+56°, 551.	II. RS Persei.	III. SU Persei.
	1909	h	m	2418				
39	Dec. 6	10	11	·647·42	52	10·31	10·85	
40	8	6	33	649·27	60	10·15	10·77	9·57
41	20	6	49	661·28	60	10·33	10·62	
42	23	9	32	664·40	60	10·05	10·47	9·55
43	31	6	20	672·26	60	10·09	10·71	9·53
	1910							
44	Jan. 7	8	47	679·37	60	10·17	10·58	
45	10	8	25	682·35	60	10·22	10·92	9·65
46	11	8	20	683·35	60	10·23	10·77	9·59
47	12	8	36	684·36	60	10·17	10·70	9·65
48	19	8	50	691·37	60	10·06	10·84	9·61
49	20	7	26	692 31	60	10·29	10·78	
50	21	8	8	693·34	60	10·24	10·97	
51	29	8	4	701·34	60	10·15	10·92	9·82
52	31	10	32	703·44	60	10·10	10·61	9·86
53	Feb. 2	8	17	705·35	50	10·15	10·65	9·87
54	3	8	31	706·35	50	10·10	10·85	9·74
55	8	7	27	711·31	50	10·11	10·75	9·84
56	10	7	14	713·30	50	10·06	10·77	9·68
57	11	9	6	714·38	27	10·26	10·96	
58	14	12	9	717·51	60	10·05	10·73	9·62
59	21	8	55	724·37	60	10·25	10·80	
60	28	10	56	731·46	60	10·17	10·92	9·59
61	Mar. 4	9	31	735·40	60	10·27	10·73	9·63
62	10	10	59	741·46	60	10·20	10·79	9·55
63	11	10	53	742·45	60	10·14	10·83	
64	15	11	4	746·46	60	10·15	10·83	9·55
65	18	10	43	749·45	60	10·22	10·71	
66	21	11	15	752·47	60	10·17	10·87	
67	29	9	8	760·38	60	9·94	10·82	9·45
68	30	8	56	761·37	60	9·94	10·74	
69	31	8	52	762·37	60	10·06	10·71	
70	Apr. 1	9	38	763·40	60	10·15	10·95	9·49
71	7	10	55	769·46	60	10·15	10·81	
72	8	10	2	770·42	60	10·10	10·92	9·48
73	9	10	9	771·42	30	10·24	10·94	9·47
74	10	10	20	772·43	60	9·99	10·77	9·49
75	16	10	53	778·45	60	9·92	10·72	9·47

TABLE I.—*continued.*

Serial No.	Date G.M.T.			Julian Day.	Exposure. min.	Magnitudes. I. B.D. +56°, 551.	II. RS Persei.	III. SU Persei.
	1910	h	m	2418				
76	Apr. 26	10	0	788·42	60	10·13	10·89	9·40
77	28	10	11	790·42	60	10·03	10·87	9·54
78	29	11	9	791·46	60	10·13	10·98	9·62
79	May 3	12	58	795·54	60	10·14	10·91	9·50
80	10	14	12	802·59	52	10·15	10·97	9·54
81	11	13	37	803·57	60	10·10	10·92	9·50
82	31	12	48	823·53	60	10·05	10·83	9·58
83	July 8	13	0	861·54	60	10·19	10·83	9·62
84	11	12	58	864·54	60	10·19	10·83	9·65
85	Aug. 9	11	57	893·50	60	10·22	10·80	9·72
86	15	10	48	899·45	60	10·17	10·53	9·76
87	17	11	57	901·50	60	9·98	10·50	9·65
88	Sept. 13	10	23	928·43	48	10·12	10·57	9·76
89	15	9	19	930·39	60	10·00	10·66	9·83
90	29	9	10	944·38	46	10·05	10·70	10·04
91	Oct 5	9	6	950·38	60	10·07	10·64	10·04
92	7	8	4	952·34	60	9·95	10·44	10·03
93	11	9	38	956·40	60	10·25	10·57	10·06
94	Nov. 1	8	12	977·34	20	10·21	10·65	10·14
95	2	8	49	978·37	60	10·04	10·47	10·06
96	4	9	34	980·40	60	10·19	10·59	
97	5	9	45	981·41	60	10·23	10·74	10·12
98	7	8	35	983·36	60	10·34	10·69	10·17
99	9	7	35	985·32	60	10·33	10·77	10·07
100	11	7	46	987·32	60	10·39	10·89	10·28
				2419				
101	28	7	58	004·33	60	10·32	10·76	9·83
102	29	7	46	005·32	60	10·52	10·60	9·67
103	Dec. 6	6	35	012·28	60	10·27	10·98	9·64
104	20	10	19	026·43	53	10·35	10·93	9·49
	1911							
105	Jan. 1	10	7	038·42	60	10·52	10·82	9·45
106	2	10	7	039·42	60	10·46	10·85	9·34
107	9	8	3	046·34	60	10·26	10·67	9·17
108	18	10	7	055·42	60	10·44	11·05	9·18
109	30	8	6	067·34	50	10·35	11·02	9·22
110	31	8	15	068·34	50	10·52	10·92	9·26
111	Feb. 23	8	19	091·35	60	10·06	10·90	9·27

TABLE I.—*continued.*

Serial No.	Date G.M.T.			Julian Day.	Exposure. min.	Magnitudes. I. B.D.+56°, 551.	II. RS Persei.	III. SU Persei.
	1911	h	m	2419				
112	Feb. 28	10	44	096·45	60	10·20	10·97	9·37
113	Mar. 6	10	54	102·46	60	10·23	10·94	9·41
114	24	9	43	120·41	60	10·39	11·09	9·53
115	25	9	57	121·42	60	10·30	10·99	9·55
116	Apr. 2	9	30	129·40	30	10·15	11·04	9·51
117	2	10	6	129·42	30	10·31	11·04	9·42
118	3	9	36	130·40	30	10·43	11·09	9·48
119	23	10	36	150·44	60	10·33	11·03	9·48
120	29	10	52	156·45	48	10·30	11·00	
121	May 3	11	46	160·49	60	10·26	11·12	9·60
122	9	13	3	166·54	60	10·25	10·92	9·52
123	18	12	19	175·51	60	10·31	10·92	9·43
124	27	12	46	184·53	60	10·01	11·10	9·51
125	29	11	47	186·49	60	10·21	11·01	9·44
126	31	11	42	188·49	60	10·16	10·95	9·45
127	July 6	12	8	224·51	60	10·27	10·92	9·48
128	Aug. 1	12	23	250·52	60	10·23	11·00	9·47
129	14	10	25	263·44	55	10·08	10·92	9·46
130	15	10	13	264·43	55	10·26	11·06	9·58
131	Sept. 14	10	35	294·44	53	10·35	11·12	9·55
132	20	9	42	300·41	60	10·55	11·11	9·62
133	21	9	30	301·40	60	10·37	11·14	9·55
134	28	8	22	308·35	60	10·17	11·25	9·56
135	Oct. 3	9	26	313·39	30	10·35	11·15	9·55
136	23	9	19	333·39	60	10·24	11·20	9·68
137	24	15	15	334·64	35	10·26	11·22	9·66
138	Nov. 2	9	41	343·40	60	10·58	11·20	9·77
139	10	8	17	351·35	27	10·38	11·00	9·90
140	16	6	37	357·28	60	10·35	10·90	9·71
141	28	16	31	369·69	35	10·28	11·05	9·88
142	30	15	53	371·66	35	10·23	10·95	9·80
143	Dec. 11	9	45	382·41	40	10·07	10·85	9·89
144	15	10	9	386·42	55	10·25	10·77	9 94
145	22	12	30	393·52	40	10·02	10·65	9·98
	1912							
146	Jan. 8	8	54	410·37	50	10·34	10·63	10·17
147	11	7	53	413·33	60	10·26	10·55	10·16
148	14	12	10	416·51	60	10·20	10·66	10·03

TABLE I.—*continued.*

Serial No.	Date G.M.T.			Julian Day.	Exposure. min.	Magnitudes. I. B.D. +56°, 551.	II. RS Persei.	III. SU Persei.
	1912	h	m	2419				
149	Jan. 30	7	6	432·30	60	10·31	10·49	9·92
150	Feb. 20	10	10	453·42	60	9·95	10·62	9·74
151	23	8	43	456·36	55	10·08	10·58	9·77
152	Mar. 6	9	4	468·38	60	9·80	10·72	9·67
153	8	9	42	470·41	60	9·94	10·85	9·68
154	15	10	14	477·43	60	10·05	10·81	9·68
155	21	8	53	483·37	30	10·18	10·87	9·76
156	Apr. 16	9	38	509·40	60	10·10	10·88	9·43
157	23	10	2	516·42	60	9·90	10·87	9·26
158	May 9	11	48	532·49	60	9·92	10·88	9·25
159	24	12	33	547·52	46	9·93	10·91	9·26
160	June 11	12	59	565·54	45	9·95	10·88	9·25
161	Sept. 5	12	15	651·51	50	10·13	11·13	
162	11	12	17	657·51	50	10·19	10·98	9·83
163	18	10	18	664·43	60	10·19	11·05	9·87
164	Oct. 7	9	58	683·42	46	10·30	11·03	10·10
165	9	9	56	685·41	60	10·36	11·06	9·88
166	31	10	17	707·43	60	10·34	11·04	9·87
167	Nov. 11	11	46	718·49	60	10·18	10·99	9·75
168	13	9	29	720·40	35	10·27	11·04	9·94
169	30	11	49	737·49	36	10·44	11·02	9·90
170	Dec. 6	8	10	743·34	35	10·28	11·04	9·92
171	9	13	22	746·56	35	10·34	11·04	9·98
172	11	13	13	748·55	35	10·26	10·87	10·04
173	16	10	14	753·43	35	10·45	11·09	10·26
	1913							
174	Jan. 1	12	38	769·53	45	10·14	10·98	10·12
175	5	12	27	773·52	45	10·35	11·05	10·12
176	Feb. 10	7	33	809·32	60	10·20	11·05	10·14

2. The photographic magnitudes given in Table I. have been derived in the same way as has been explained in recent papers published by this observatory on SU and SW Draconis.* From the investigation of the earlier part of the present series of plates, with the help of the photographic magnitudes determined by Schwarzschild for stars of the χ Persei cluster,† Professor Whittaker concluded that the formula connecting magnitude

* *M.N.*, lxxiii. pp. 166, 440.
† *Publ. der v. Kuffner'schen Sternwarte*, V. C., p. 81.

with measured diameter which best suited the Dunsink plates was

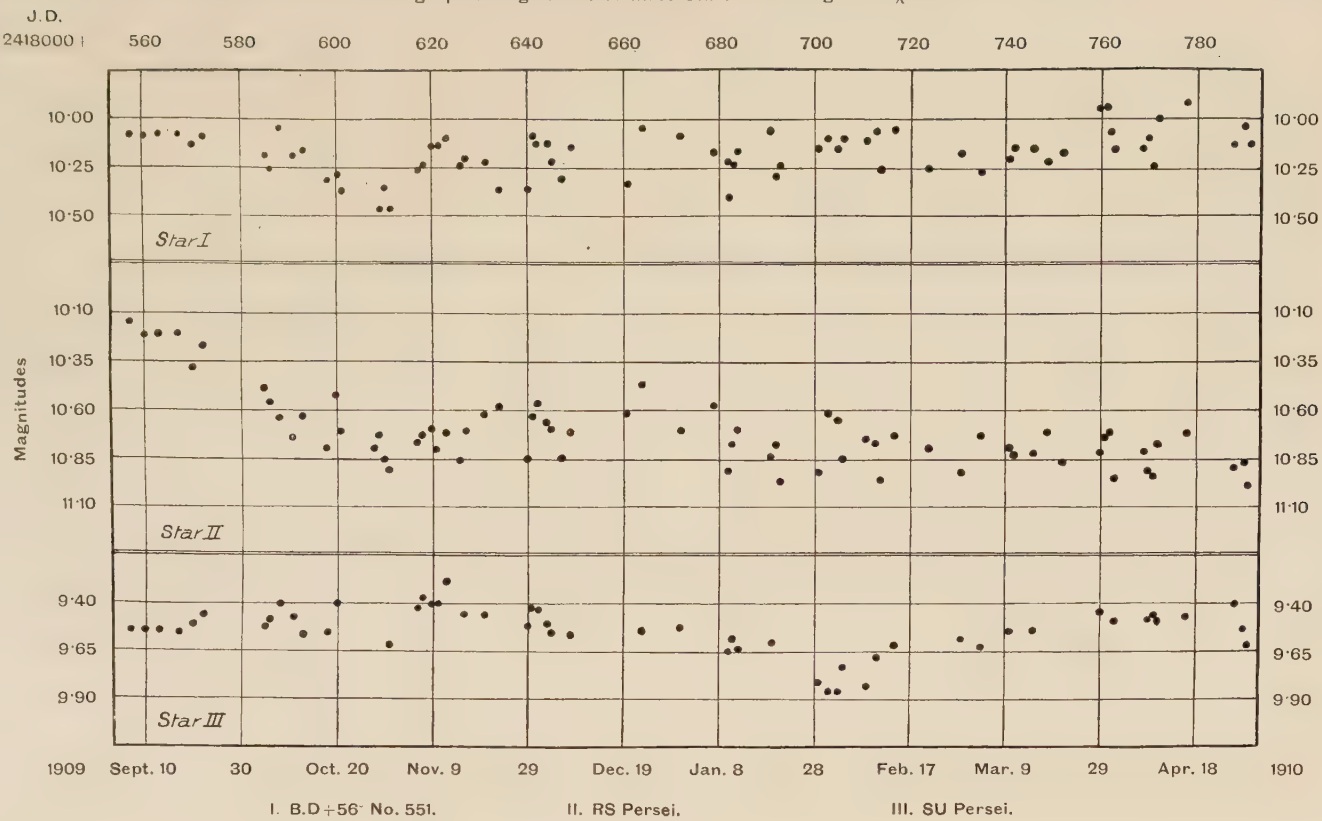
$$m = a - b\ D^{\frac{2}{3}}.$$

The term "measured diameter" must here be strictly understood in the sense explained in the papers quoted. This formula being assumed, the constants a and b are determined for each plate by the assumed magnitudes of certain comparison stars. The information contained in Table II. refers to the five comparison stars used in deducing the magnitudes of the variables I. and II., which lie fairly near the centre of the plates. The third variable, SU Persei, lies at a distance from the other two and has been treated independently throughout in a way which will be explained later. The magnitudes of the five stars in Table II. had been determined by Schwarzschild, but the provisional magnitudes here given are adjusted values derived by Professor Whittaker.

TABLE II.

Star.	B.D.	Provisional Magnitude.	Range.	P.E.	Mean Correction.	Final Magnitude.
	°		m	m	m	
e	+56·557	9·98	·15	±·015	+·009	9·99
f	+56·590	10·04	·14	·013	−·009	10·03
g		10·52	·33	·037	−·027	10·49
h		10·78	·18	·018	−·009	10·77
i		11·00	·17	·021	+·010	11·01

3. As in the previous cases quoted, the method by which the magnitudes of the variables have been obtained has been a graphical one. For the comparison stars the points ($D^{\frac{2}{3}}$, m) have been plotted and a straight line drawn to fit them as closely as possible. For the variables the abscissæ $D^{\frac{2}{3}}$ are known, and the corresponding ordinates of points on the line are read off and give the magnitudes which are to be found in Table I. The deviations of the line from the points which represent the comparison stars are also read off, and supply a set of residuals for each comparison star. The sum of the greatest excess and the greatest defect is given in the column headed "Range" in Table II. Similarly the mean of these sets of residuals is given in the column headed "Mean Correction." The next step is to apply these corrections to the residuals, and thus a second set of residuals is obtained. These are then taken positively, and their simple means, diminished by one-sixth part, are given in the column headed "P.E." This summary method of calculating the probable error undoubtedly gives values which are too small, for two reasons: because the mean of the errors is known to minimise the effect of large residuals, and because the interpolated line adapts itself to the errors of the points on which it is based. The method, however, is quick, and for


Photographic Magnitudes of three Stars in the Region of χ Persei.
J.D.
2418000 +
560
580
600
620
640
660
680
700
720
740
760
780
Magnitudes
10·00
10·25
10·50
Star I
10·10
10·35
10·60
10·85
11·10
Star II
9·40
9·65
9·90
Star III
1909
Sept. 10
30
Oct. 20
Nov. 9
29
Dec. 19
Jan. 8
28
Feb. 17
Mar. 9
29
Apr. 18
1910
I. B.D+56° No. 551.
II. RS Persei.
III. SU Persei.

merely comparative purposes it gives results of the right order of magnitude. The last column in Table II. gives the final magnitudes of the comparison stars, found by applying the mean corrections to the provisional magnitudes. The adjustment is so slight that it is unnecessary to apply any correction to the magnitudes of the variables. Probably the star *g* is itself slightly variable.

4. Star III.—the variable SU Persei—is situated near the southern edge of the region photographed. It has therefore been treated quite independently. The information relating to the four surrounding comparison stars is given in Table III. This is in the same form as Table II., and similar remarks apply to it, with one exception. The provisional magnitudes are not in this case adjusted values based on another source, but have been determined from a pair of plates on each of which were photographed the χ Persei cluster centrally and also the region of SU Persei brought to the centre of the plate. The comparison magnitudes were thus based on the χ Persei stars in the way which has been explained in former papers, and the provisional magnitudes given in the table are the means of the results obtained from the two plates. In spite of the fact that the interpolated line is based on four instead of five points, the probable errors are larger in Table III. than in Table II. if we except the star *g*, suspected of variability. Hence we infer that one factor in securing the greatest accuracy in an individual magnitude depends on the proximity of the star to the centre of the plate.

Table III.

Star.	B.D.	Provisional Magnitude.	Range.	P.E.	Mean Correction.	Final Magnitude.
	°	m	m	m	m	
w	+55·600	9·34	·22	±·023	−·014	9·33
x	+55·596	9·52	·19	·022	+·008	9·53
y	+55·594	9·80	·26	·029	+·029	9·83
z	+56·569	10·56	·20	·022	−·024	10·54

5. The diagram (Plate 17) which accompanies this paper exhibits the results of the observations in the season 1909–10. It will be seen that all the three stars show the same type of irregular variation. No attempt is made to elucidate further the nature of these fluctuations, and indeed for this purpose it would be necessary to possess observations much more continuous and connected in point of time. To secure such observations would be possible only at a station more favourably situated than Dunsink in respect to climate. The observation of these stars has now been discontinued here. It may be remarked that the difference between the visual and photographic magnitudes shows that the three stars are all distinctly red.

6. Under present circumstances it is impossible to describe in detail the course of the earlier researches, largely experimental,

which were based on this series of plates. Nor is it necessary. The number of stars measured on each plate was gradually reduced from the beginning of the work, but eleven stars were retained until 1911 May 3, since which date only the five stars to which Table II. refers were employed. It is to be understood that the variable stars and the four comparison stars of Table III. are additional to these. The plates here numbered 1–79 were reduced numerically by the formula, those numbered 80–121 by the graphical process described. But no distinction has been or need be drawn between the residuals derived by the two methods. For different reasons the results for a few of the plates have not been used. The number of plates is given in Table IV., and it will be noticed that the two faintest stars have been measured on scarcely more than half the plates. The magnitudes of Schwarzschild are given in one column, and the provisional magnitudes resulting from Professor Whittaker's adjustment are given in another. The discrepancies between the two columns may be due, at least in part, to the different type of instrument employed. The probable errors and mean corrections have been calculated as in Table II., and the resulting final magnitudes are given in the last column. The results for the five stars common to Tables II. and IV. are based on the same measures, but the reductions are quite independent except for the adoption of the same set of provisional magnitudes.

TABLE IV.

Star.	B.D.	No. of Plates.	Magnitude (Schwarzschild.)	Provisional Magnitude.	P.E.	Mean Correction.	Final Magnitude.
					m	m	
a	+56°568	116	7·21	7·20	±·040	−·002	7·20
b	+56·593	,,	7·71	7·81	·033	−·055	7·75
c	+56·591	,,	8·35	8·27	·040	+·001	8·27
d	+56·588	,,	9·31	9·10	·049	·000	9·10
e	+56·557	,,	9·97	9·98	·035	+·017	10·00
f	+56·590	,,	10·07	10·04	·038	−·013	10·03
g		115	10·57	10·52	·038	−·020	10·50
h		116	10·65	10·78	·037	−·008	10·77
i		,,	10·83	11·00	·039	−·001	11·00
j		76		11·33	·053	+·045	11·37
k		65		11·81	·076	+·006	11·82

7. It will be noticed that the final magnitudes in the two tables differ in no case by more than $0^m{\cdot}01$. But the probable errors are quite different. It was thought that some cause for this was to be found in the greater changes of the light in the course of measuring a greater number of stars and perhaps in the fatigue of the measurer. This idea, however, can scarcely be entertained, because the measures of the stars *e*—*i* are the same in

both cases except for the later plates utilised in Table II. In part, the effect can doubtless be attributed to the greater power of accommodation of the interpolated line to the errors of five points as compared with the correspondence between a line or linear formula and eleven points. But there is probably an additional reason, connected with the greater range of magnitude. The adopted formula which expresses the relation between diameter and magnitude may be regarded as perhaps the best under mean conditions. As the circumstances vary from plate to plate, in respect to emulsion, conditions of exposure and process of development, it is conceivable that the relation also varies slightly, not merely as regards the two constants which enter into it but in functional form also. Then, for any particular plate, the relation between m and $D^{\frac{2}{3}}$ is only approximately linear, and when the total range of magnitude is considerable errors will arise from this cause alone. On the other hand, when the range of magnitude is very small the functional form of the relation is relatively unimportant and the linear formula is accurate enough. Hence it is regarded as a matter of importance in the determination of the magnitude of a particular object that the range of magnitude in the standards of comparison shall not be much greater than may be required to cover the range of variation of the object itself.

8. It may be noticed that the variability of the star *g* which is suspected on the evidence of Table II. is masked among the larger residuals which are summarised in Table IV. On the other hand, the largest probable error among the brighter stars of the latter table is shown by the star *d*. The measurer noticed in this case that the image of the star was of distinctly irregular shape affecting the accuracy of the measures. It has been verified subsequently that this star is A.G. Hels. 2171 = Aitken 963, a triple system, the magnitudes of the two companions being 13·0 and 13·5. The most striking deduction to be drawn from Table IV. is conveyed by the probable errors for the two faintest stars. These show clearly that the general order of accuracy, which is well maintained as far as the eleventh magnitude, rapidly deteriorates after this point has been reached. This must be considered in conjunction with the fact that the plates have been taken with a 15-inch reflector and with exposures, in most cases, of one hour. The limiting magnitude for accurate results must depend upon the instrument and the time of exposure, but it is concluded that this limit is comparatively high—very considerably above the limiting magnitude recorded on the plate.

9. The following places (Table V.) for all the stars mentioned in this paper are derived from different sources which need not be specified because they are given merely in order to ensure accurate identification without reference to memoirs which are not always available.

TABLE V.

Star.	Phot. Mag.	R.A. 1900·0.	Decl. 1900·0.
		h m s	° ′ ″
I.	var.	2 13 56·39	+56 41 54·4
h	10·77	14 10·13	40 54·8
e	9·99	14 13·70	41 9·5
y	9·83	14 49·04	7 1·8
a	7·20	14 51·22	47 4·5
z	10·54	14 52·67	15 10·9
g	10·49	15 0·93	43 7·6
x	9·53	15 2·60	3 57·5
III.	var.	15 5·30	8 45·8
II.	var.	15 20·36	39 6·1
i	11·01	15 26·63	46 43·8
d	9·10	15 39·41	37 46·4
f	10·03	15 44·49	41 48·3
c	8·27	15 48·56	47 17·2
w	9·33	15 52·01	9 59·0
b	7·75	15 54·79	55 48·5
k	11·82	16 1·30	39 32·1
j	11·37	2 16 12·12	+56 40 42·3

10. In addition to the observations of three variable stars in the neighbourhood of the χ Persei cluster, the main outcome of certain subsidiary researches has been included in this paper. The chief conclusions may be summarised thus:—

(1) The star whose magnitude is to be studied should be situated near the centre of the plate, with a uniform distribution of the comparison stars if possible.

(2) The comparison stars should furnish a restricted light scale not much exceeding in range what is required to cover the variation of the object.

(3) The time of exposure should be such that the object is considerably brighter than the limiting magnitude recorded on the plate.

These conditions are all more or less incompatible with the requirements of general astronomical photometry, in which it is necessary to investigate stars of widely different magnitudes over a considerable area of the plate. To this should be added that with the images formed by the Dunsink reflector it has been found possible to regard them as discs of varying density, and to make the settings not on the edge but at the point of maximum density gradient. These considerations may help to explain the apparent accuracy of the Dunsink results, which, though distinctly flattered by the probable errors given for the reasons alluded to above, ought to be superior to that attainable under less specialised conditions. There is, again, one reason for suspecting that the order of accuracy in general photometric work may well be underestimated. The estimate is usually based on the comparison of many stars but few plates of a particular region. There is, therefore, little opportunity for detecting and eliminating those stars

which are variable. On the other hand, experience at Dunsink based on the comparison of few stars but many plates of a single region has suggested (though the material is meagre) that 20 per cent. of the stars vary by some tenths of a magnitude. This may account for some of the larger discrepancies which are known to have presented themselves in the determination of photographic magnitudes on the larger scale.

Dunsink Observatory:
1914 *June* 8.

The Magnitude of η Argûs, 1914, *and discovery of a close companion to it.* By R. T. A. Innes.

The comparison stars were, as in former years (see *Monthly Notices*, lxix. p. 632)—

C.G.A. Cluster Catalogue No. 121,	8·0 mag.	8 colour.
Gilliss 1332,	7·6 ,,	4 ,,

The separate observations are:—

1914 May 10	7·8 mag.	6 colour.
,, 11	7·7 ,,	6·5 ,,
,, 12	7·9 ,,	7 ,,

The observations were made at the 9-in. Grubb refractor, using a small hand-map.

The means by years of all the observations made by myself are:—

	Mag.	Colour.	No. of Obs.
1896·4	7·58	5·0	4–1
1899·5	7·71	7·0	4–2
1900·3	7·68	6·3	3
1901·3	7·78	6·8	3
1902·1	7·72	6·5	3
1905·5	7·67	7·5	5–2
1908·4	7·75	5·7	4–3
1909·3	7·78	5·7	3
1914·4	7·80	6·5	3·

These observations show no variation since 1899 greater than the errors of estimation.

The observations made this year were prompted by the footnote on the first page of *Lick Observatory Bulletin*, No. 252, in which it is mentioned that it was reported in some newspaper that η Argûs was visible to the naked eye in 1913. This report is certainly erroneous, as, although special observations were not made here in 1910–1913, a frequent (almost nightly) visual watch is made and no change was seen. Further, unless the star got very bright (brighter than 4·0 mag.), the region is so crowded with stars that its identification with the naked eye would be impossible.

At the first glance on the 10th inst., it was plain that η Argûs was not a single star, and the use of a higher power showed that

it had a faint companion north following (74°, 1″, mags. about 8·0 and 10·5): on account of the difference of magnitudes it would be classed as a moderately difficult pair. Although it has not been my habit to make a record of stars examined and found single, I am sure that on more than one occasion I have carefully examined the region for double stars. I believe I did so with the 18-in. McClean refractor about 1900, and found the star single. There is no doubt that Professor See also unsuccessfully examined the star for duplicity in 1897, as his Nos. 122 and 123 are near it. Hence there is a fair *a priori* probability that the companion is in orbital movement, with a suggestion that the outbursts of light which have occurred in the past have been caused by periastral grazings.

Both Mr. Wood and Mr. Worsell saw the companion readily, and the former remarked that η Argûs was fuzzy. This I was able to confirm easily; it is impossible to focus η Argûs sharply, whilst neighbouring stars of much the same hue, as well as those both redder or yellower, can be sharply focussed.

Union Observatory, Johannesburg.

Discovery of a Variable Star in Carina. By H. E. Wood.

On comparing together two plates taken respectively on 1911 April 18 and 1914 April 27 with the aid of the Zeiss Stereocomparator, I found that the star C.P.D. – 59°, 2860 : $10^h\ 51^m\ 10^s{\cdot}5$, – 59° 47′·1 (1875), mag. 7·64 was variable. The magnitudes of the star on the two plates were roughly 6·5 and 8·0, the star being brighter on the earlier photograph. This star does not appear in the A.G. Catalogue of Variable Stars, but is near to the two variables T and U Carinæ. The images of U Carinæ showed a distinct difference of magnitude of about the same order and in the same direction as those of C.P.D. – 59°, 2860. With reference to the star T Carinæ, there seems to be some considerable uncertainty. The position of this star is $10^h\ 50^m\ 18^s{\cdot}0$, – 59° 51′·1 (1875). Careful examination of the two plates showed no difference between its two images, so that either the star was not variable or it was photographed at similar epochs. However, as the variability of this star has not been confirmed (Harvard revised Photometry), it appears now to be quite certain that Gould's star is not variable, but it is possible that there has been a misidentification.

I sent my observations to Dr. A. W. Roberts of Lovedale on April 30, who, in reply, states that the problem of T Carinæ is now probably solved. Dr. Roberts has one or two old observations of C.P.D. – 59°, 2860 which, although not of much value, indicate change of some kind. He has placed the star on his observing list, and is observing it on every possible night.

Union Observatory, Johannesburg.

Nebulæ seen on Mr. Franklin-Adams' Plates. By J. A. Hardcastle, with Note by A. R. Hinks. (Plate 18.)

In the *Monthly Notices*, vol. lxxi., No. 7, Mr. Hinks threw out the suggestion that it would be an advantage if the series of 206 plates on which Mr. Franklin-Adams photographed the whole sky could be examined and a statement made of the nebulæ visible on them. By the kind permission of the Astronomer Royal these plates have been placed at my disposal, and I now present the result of a scrutiny carried out simply with a view of obtaining a list of the more conspicuous objects found on them. The position angles of those that are markedly elongated have also been added. The aperture of the instrument was 10 inches and the focal length about 4 feet, so that $1^\circ = 2$ cm. The exposures were usually two hours.

The following classification has been adopted :—

Class I.—Spirals.

This class includes :—

(*a*) 33 nebulæ which clearly show a spiral structure.
(*b*) 87 nebulæ which were judged to be probably spiral. About 20 of these are known from other sources to be spirals.
(*c*) 53 nebulæ of which it was impossible to state the structure, but which are known already to be spirals.

These 173 objects are plotted on fig. 1.

Class II.—Elongated.

These are spindle-shaped, with or without a central condensation or nucleus ; some are oval, with length at least double of the breadth. These 233 objects are plotted on fig. 2.

Class III.—Diffused.

This class contains the large gaseous nebulæ already well known and any others of which the images on the plates exceeded 2′ in diameter, but show no structure. Of these there are 51.

Class IV.—Small.

Of these all that can be said is that they are distinguishable from star-images on these plates. About 20 which were originally placed in this class were found to be known to be spiral and were transferred to class I (*c*). About 25 others which appear in the lists given by Mr. Hinks in *M.N.*, vol. lxxi., No. 8, have also been removed, and the remaining 327 have been plotted in fig. 3.

The position angles which are printed in italics differ notably from those given in the *N.G.C.*

CLASS I(*a*). 33 SPIRALS.

N.G.C.	Pos. Ang.	N.G.C.	Pos. Ang.	N.G.C.	Pos. Ang.
224		1300	99°	4254	
246		1365		4303	100°
247	172°	1566		4321	127
300		2442	39	4536	120
598		2903	8	5194	
613		2997		5236	
628		3031		5247	
908	72	3627		6744	
1097	144	3631		7125	90
1187		3893	167	7424	
1232		4051		7479	10

The following have not, I believe, been described before :—
300 !! 15′ × 20′, B no N, B * inv.
1097 !! eBN, arms ext. to 6′ × 8′, central spindle 6′ in 144°.
1232 R. BN. 6′ 2 arms.
2442 !! F. 2 arms 6′ 39° cFN.
7424 cF. R. 6′.

CLASS I(*b*). 87 ?SPIRALS.

N.G.C.	Pos. Ang.	N.G.C.	Pos. Ang.	N.G.C.	Pos. Ang.
55	108°	1510		3359	1°
134	41	1518	30°	3511	68
150		1527	88	3621	153
157	27	1531	24	3623	11
210	27	1532		3646	132
428		*1784		3649	
450		1808	144	*3686	
578	130	*2146	125	*3718	
672	72	2207	*109*	3794	
894		2280	158	4027	
895		2403		4116	155
925		2532	5	4123	*130*
936		2835	179	4258	147
986		3079	159	4364	
1042		3115	40	4487	
1313	0	3169	80	4490	
1385	0	3175	52	4501	142
1398		3223	142	4517	85
1406	17	3281		4527	50
1425	129	3354	139	4535	

* 1784, 2146, 3686 described as Sp. in *L.O.B.*, 248. 3718 in *L.O.B.*, 248, is Bifid irreg. 3′.

CLASS I(*b*). 87 ? SPIRALS—*continued.*

N.G.C.	Pos. Ang.	N.G.C.	Pos. Ang.	N.G.C.	Pos. Ang.
4559	137°	*5427		7412	
4569	10	5468		7456	18°
4579		†5927		7496	
4605		6907		7552	90
4654	124	‡7023		7599	
4945	39	7083	13°	7723	
5068		7107		7756	
5128	121	7141	9	7793	102
5334		7184	46		
5364		7410	32		

In II. Index Catalogue 2132, 132°, and 5332.

CLASS I(*c*).

221	3596	4402	5033
246	3810	4406	5897
488	3938	4449	6217
1068	3991	4450	6300
1433	4030	4472	6643
1559	4062	4485	6872
1637	4088	4548	7217
1672	4136	4571	7293
1792	4150	4618	7329
3034	4189	4631	7814
3109	4214	4639	II. 2995
3368	4236	4651	
3423	4374	4725	
3556	4382	4826	

CLASS II.

N.G.C.	Pos. Ang.	N.G.C.	Pos. Ang.	N.G.C.	Pos. Ang.
25	39°	493	50°	1055	*90*°
131	51	520	140	1084	34
185	35	625	98	1233	69
205	160	645	130	1249	71
253		779	155	1332	108
289	163	891	*10*	1350	13
337	130	958	0	1353	132
406	25	1003	89	1360	33

* D.S. says "probably ring."
† ⊕ Δ.
‡ Crossley 65.

CLASS II.—*continued.*

N.G.C.	Pos. Ang.	N.G.C.	Pos. Ang.	N.G.C.	Pos. Ang.
1428		3549	*27*°	4208	51°
1448	29°	3571	90	4216	16
1507	5	3593	91	4217	41
1511	116	3628	*74*	4220	136
1515	14	3669	156	4224	40
1553	143	3673	77	4235	46
1558	58	3675	178	4244	42
1560	10	3705	47	4246	75
1602	76	3726	1	4274	97
1617	122	*3729	153	4293	56
1620	*20*	3735	135	4294	141
1771		3756	179	4298 }	131
2543		3769	140	4302 }	
2613	115	3877	30	4304	
2715	10	3917	57	4307	22
2770	145	3949	133	4312	157
2784	59	3953	10	4313	136
2815	9	3957	172	4314	143
2841	144	3972	118	4350	27
2976	150	3976	45	4388	92
3020	*93*	3981	16	4389	93
3024	123	4010	54	4414	164
3044	115	4038	45	4419	132
3054	142	4064	150	4429	97
3187	116	4094	50	4448	94
3190	116	4096	*15*	4460	*34*
3198	30	4100	155	4469	87
3227	152	4109		4480	0
3254	35	4124	110	4498	132
3309	160	4125	77	4502	131
3347	179	4134	140	4504	150
3365	157	4144	101	4522	31
3368	33	4169		4526	111
*3389	83	4178	25	4546	85
3412	*13*	4179	*150*	4565	134
3449	143	*4192	136	4567	62
3454	44	4197	30	4568	15
3521		4206	1	4586	120

* *L.O.B.*, 248, spiral.

CLASS II.—*continued.*

N.G.C.	Pos. Ang.	N.G.C.	Pos. Ang.	N.G.C.	Pos. Ang.
4592	90°	5577	50°	7582	135°
4594	87	5584	150	*7640	
4602	100	5595	38	7721	10
4634	143	5676	38	7817	40
4656 }		5690	145	I. 749	
4657 }		5728	22	I. 1151	24
4666	45	5866	119	II. 1627	133
4698	158	5900 }	20	II. 1657	0
4710	18	5901 }		II. 1783	0
4712	167	5984	148	II. 1954	31
4746	108	6015		II. 1970	70
4747	37	6293		II. 2469	32
4762	21	6707	151	II. 2531	66
4793	48	6769		II. 2554	11
4835	150	6887	100	II. 2555	39
4856		6925		II. 2974	95
4866	89	6943	132	II. 3253	27
5005	55	7038		II. 4720	164
5012	10	7059	88	II. 4721	154
5055	108	7064	97	II. 4810	
5102	37	7090	126	II. 4837	
5161	66	7098	39	II. 5039	
5170	310	7124	142	II. 5041	
5248	125	7162		II. 5052	141
5253	35	7179	16	II. 5152	109
5348	175	7218	30	II. 5171	158
5356	15	7307	2	II. 5176	25
5483	13	7331	153	II. 5249	6
5496	0	7361	9	II. 5269	102
5507	90	7448	169	II. 5270	102
5529	117	7462	73	II. 5271	137
5560	105	*7497	25	II. 5273	37
5566	150	7531	16		

There is also a nebula at R.A. $13^h\ 5^m$ N.P.D. 121°53′ not in the *N.G.C.*, cF. $3' \times \frac{1}{2}'$ 154°.

* *L.O.B.*, 248, spiral.

Class III.

346		Diffused	2736		Diffused
521		,,	3113	90°	,,
864		,,	3137	0°	,,
1435		Merope	3324		,,
1549		Diffused	3351		M. 95
1574		,,	3372		η Argûs
1760/1		Magellanic Cl.	3587		The Owl
1783		,,	3672	178°	Diffused
1788		,,	4050	90°	,,
1952		Crab	4532	160°	,,
1976		θ Orionis	5085		,,
2011		Magel. Cl.	5339		,,
2023	140°	ζ Orionis	6503	125°	,,
2024		Diffused	6514		Trifid
2030		Magel. Cl.	6618		The Omega
2068	15°	Diffused	6726/7		Diffused
2069/70		Looped Neb.	6822		,,
2103		Magel. Cl.	6853		The Dumb-bell
2237		Diffused	6992		Cygnus
2257		,,	7293		Helix (Shaw)
2264		,,	7314	0°	Diffused
2417		,,	II. 1727		,,
2427	135°	,,	II. 2177		,,
2467		,,	II. 2944		,,
2517		,,	II. 4499		,,
2626		,,			

Class IV.

121	458	881	1316
132	514	897	1344
147	516	922	1367
152	602	941	1395
173	643	942	1466
234	685	950	1473
238	701	1022	1533
278	720	1199	1536
339	753	1208	1543
361	755	1209	1546
404	782	1302	1581
411	788	1309	1596
419	852	1315	1629

Class IV.—*continued.*

1651	2177	3336	4106
1652	2187	3358	4143
1676	2190	3367	4152
1703	2203	3370	4158
1718	2209	3377	4168
1736	2213	3379	4193
1779	2214	3384	4215
1789	2249	3414	4241
1796	2305	3430	4257
1805	2389	3455	4260
1806	2397	3464	4272
1826	2424	3485	4273
1831	2434	3486	4278
1852	2573	3488	4310
1884	2633	3489	4336
1887	2665	3504	4351
1900	2763	3512	4365
1920	2787	3513	4371
1944	2849	3514	4380
1951	2855	3546	4407
1978	2889	3557	4430
2010	2916	3564	4435
2046	2943	3568	4442
2047	2968	3607	4452
2053	2971	3608	4458
2075	2983	3626	4459
2082	2985	3655	4461
2105	3001	3659	4465
2107	3019	3660	4473
2111	3059	3681	4474
2114	3077	3684	4476
2120	3091	3813	4479
2121	3095	3887	4494
2133	3162	3923	4496
2145	3166	3941	4503
2147	3193	3973	4519
2154	3208	4026	4525
2155	3226	4036	4531
2162	3268	4037	4540
2166	3275	4074	4550
2173	3285	4105	4551

Class IV.—*continued.*

4552	5230	6780	I. 1303
4561	5300	6848	II. 1933
4570	5324	6878	II. 2056
4574	5327	6880	II. 2070
4578	5363	6902	II. 2073
4580	5419	6935	II. 2113
4589	5494	6937	II. 2202
4593	5576	7006	II. 2440
4596	5597	7007	II. 2522
4606	5633	7070	II. 2537
4621	5645	7126	II. 2580
4633	5665	7135	II. 2582
4647	5668	7192	II. 2588
4689	5674	7321	II. 2627
4693	5695	7377	II. 2631
4696	5701	7457	II. 3026
4697	5893	7515	II. 3115
4733	5967	7590	II. 3797
4750	5970	7689	II. 4628
4753	6060	7755	II. 4662
4754	6077	I. 59	II. 4830
4902	6102	I. 63	II. 4901
4947	6106	I. 382	II. 5156
4958	6235	I. 391	II. 5186
4965	6245	I. 405	II. 5265
4995	6340	I. 532	II. 5325
5042	6355	I. 602	
5172	6753	I. 724	

For all the heaviest part of the work, such as identifying the *N.G.C.* numbers and sorting out the various classes, I am indebted to Miss A. G. Cook, for whose most efficient help I wish to express my thanks.

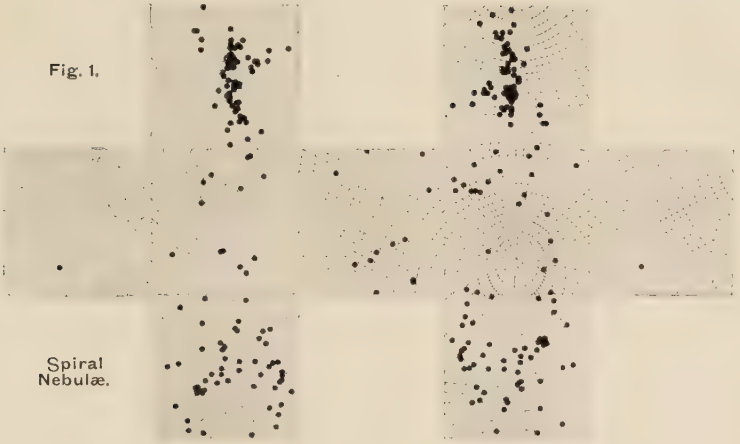
Crowthorne, Berks.:
1914 *June.*

Note by A. R. Hinks.

The author of this paper kindly allows me to add a note on his results.

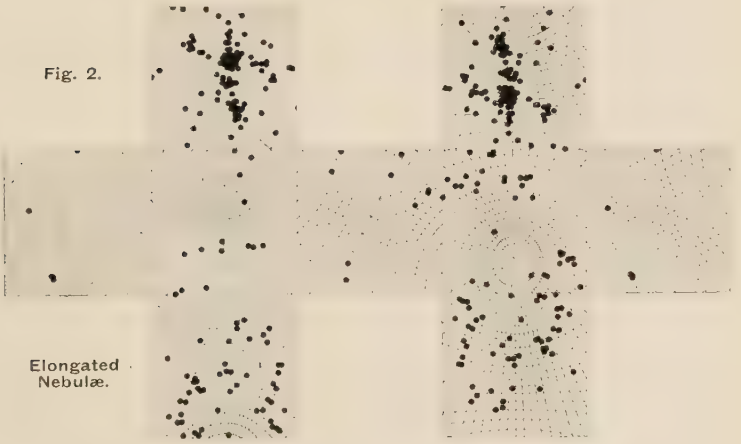
We have now available for the first time a count and rough classification of nebulæ, from plates covering the whole sky, made with one instrument, under fairly uniform conditions. Only a small proportion of all the known nebulæ are recognisable on the plates; but as far as they go the results are unbiased. In

Fig. 1.



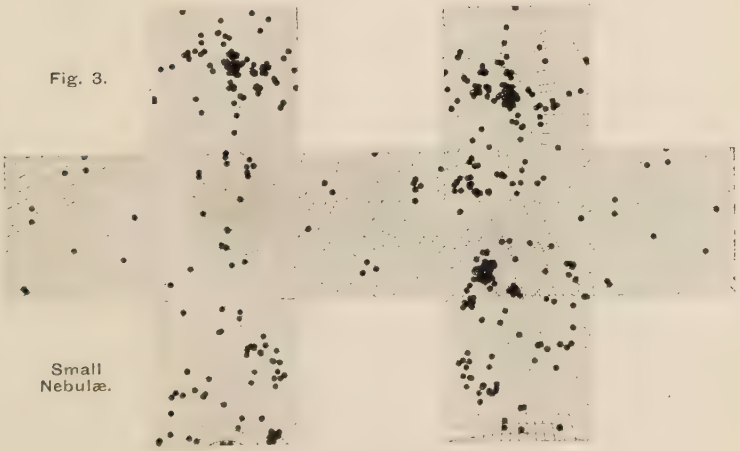
Spiral Nebulæ.

Fig. 2.



Elongated Nebulæ.

Fig. 3.



Small Nebulæ.

particular they are not affected by the tendency to search in regions known to be rich.

The results have been plotted on the projection used in my papers in *M.N.*, vol. lxxi., Nos. 7 and 8, and may thus be compared readily with my earlier diagrams. If we letter the squares containing the northern and southern galactic poles N and S, and the galactic squares A, B, C, D, we have the following numbers:—

	Spirals.	Spindles.	Small.
N	70	118	122
S	52	51	58
A	1	4	10
B	13	14	18
C	12	9	13
D	23	34	106

It should be noted that the nebulæ here called small are not so small as the smaller planetary and the stellar nebulæ. The latter are indistinguishable from stars on these plates.

The conclusions which I would draw are briefly as follows:

1. Open spirals are nearly equally common in the northern and southern squares, but there is a great excess of spindles in the northern, as shown in column 2 and reflected in column 3. This excess of spindles in square N is the most important fact established by Mr. Hardcastle's investigation.

2. The notable absence of spirals and spindles in square A, previously remarked by me, is confirmed by this paper. Along the other galactic squares the distribution approaches uniformity, except for square D, "small," which is influenced by the Magellanic cloud, whose nebulæ are of a quite different kind.

3. The old view that the distribution of these spirals and spindles, the "white" nebulæ, is determined by mere avoidance of the galactic plane cannot be maintained. Whatever may be the law of distribution, it is not a function of galactic latitude only. Galactic longitude is equally important.

Note on the Star 42 Virginis. By E. W. Barlow, B.Sc. (Plate 19.)

The star in the constellation Virgo to which the Flamsteed number 42 was allotted is now apparently absent from the sky. There is no object in or close to its position except two stars of the 9th and one of about the 10th magnitude. The disappearance of the star 42, of about the 6th magnitude, is alluded to in several astronomical works of the nineteenth century, and is left unexplained. Furthermore, the evidence for and against such an object ever having existed is presented in a very partial and incomplete form. This evidence I have attempted to render as complete as possible,

and I propose to systematise it in the present paper. The whole matter seems to have been forgotten, but is nevertheless of great interest as throwing light on the possibility of the existence of types of stellar variation of different character from those which have been deduced from the study of the ordinarily recognised variable stars.

It is possible that the matter was considered at an end after Sir John Herschel's footnote in the Cabinet Cyclopædia edition (1851) of his *Outlines of Astronomy*. To the description of the phenomenon he then appended these words:—"This has since been accounted for. 1851." It is unfortunate that Sir John, after describing this star, should have given no indication of the nature of the explanation.

The evidence which I shall set forth, however, seems to dispose of any possibility of explanation other than the visibility of the star at certain epochs and its non-visibility at others, and there is at least one definite observation of it since the above remark was made, viz. by Houzeau in 1875.

In Baily's edition of Flamsteed's British Catalogue these particulars occur:—No. 1785, Magn. 6, R.A. 189° 49′ 20″, Decln. +9° 30′ 30″, epoch 1690. To this star Baily appends these remarks:—"I cannot find any observation of this star in Flamsteed's *Historia Cœlestis*; nor can I discover that such a star exists. There is a star of the 7th magnitude in Zach's Catalogue, in the first volume of his *Tabulæ Speciales*, which is called 42 Virginis, and which was seen both by him at Seeberg and by MM. Barry and Henry at Mannheim. The Declination of this star agrees very well with the British Catalogue, but the R.A. differs upwards of 3′. It is remarkable that this star is also not now to be found. For although Professor Airy has at my request looked out for it and found a star agreeing with that of Baron Zach in R.A., yet the Declination differs more than 4′ from that given by MM. Barry and Henry; and is, moreover, only of the 10th magnitude. Neither Piazzi, Lalande, or Bessel has any star that can be mistaken for it."

Baron Zach in his *Tabulæ Speciales*, vol. i., gives the following data:—No. 886, Virgo, 42 Flamsteed, Magnitude 7, R.A. for 1800 $12^h\ 45^m{\cdot}1$. Declination at beginning of 1800 + 8° 54′ 23″. Admiral Smyth in his *Cycle*, vol. ii. p. 290, says that "De Zach's star was regularly observed and registered."

This star is found in the Astronomical Society's Catalogue, epoch 1830, the position given being identical, allowing for precession. Sir John Herschel states clearly that this insertion is on the authority of Zach.

So far we have definite observations of a star of the 7th magnitude by three observers, Zach, Henry, and Barry, at some date in the neighbourhood of 1800, which star is non-existent to-day. Flamsteed's observation might from Baily's remarks be considered an error, clerical or otherwise; it would seem feasible to suppose that Flamsteed observed no such star, especially as its position, according to Baily, does not agree with that of Zach. Convert-

ing Flamsteed's R.A. to hours, we have $12^h\ 39^m\ 17^s{\cdot}3$, epoch 1690; and using the precession figures given in the Astronomical Society's Catalogue (viz. $+3^s{\cdot}027$ in R.A., $-19''{\cdot}630$ in Decln. annually) we find Flamsteed's position for 1800 to be $12^h\ 44^m\ 50^s{\cdot}28$, $+8°\ 54'\ 30''{\cdot}8$. There is hence the discrepancy that Baily states; the difference is $15^s{\cdot}7$ in R.A., *i.e.* between 3′ and 4′, and about 8″ only in Decln.

It seems probable, however, that Flamsteed did really see this object, and that his R.A. was in error, or perhaps there was an approximate observation only. If not, it would be a curious coincidence that the position of the imaginary object should lie so close to that of the star observed by Zach and others.

It should be noticed that the nearest bright star to the place under discussion is D.M. $+9°$ 2696 (P XII 227), which lies between δ and ϵ Virginis and is of 7·4 magnitude according to the D.M. This star lies about 41′ *nf* the place of 42.

Fortunately there is evidence that 42 Virginis was observed between 1738 and 1745. The evidence in question is the existence of this star in Dr. Bevis's remarkable and original atlas *Uranographia Britannica*, a very few copies of which are extant. We may assume that this atlas is not merely a copy of Flamsteed, or made from his catalogue, by reason of the following particulars as to Bevis's life extracted from the *Dictionary of National Biography* and Hutton's *Mathematical Dictionary*. Dr. Bevis in 1738 erected an observatory at Stoke Newington, where he made such numerous transit observations, often 160 in a single night, that by 1745 he had obtained sufficient materials to compile his work, which purported to be an exact view of the heavens, including many more stars than had been given in Bayer's maps. A catalogue accompanied the atlas, but owing to the bankruptcy of the publisher of the work the catalogue was never published, and only a very small number of copies of the atlas were produced. Dr. Bevis gives the star as magnitude 6, the same as Flamsteed, but it seems certain that his magnitudes as well as his positions are original, since they do not in many cases agree with those of Flamsteed.

The next observation of 42 is an absolutely decisive one, about which no mistake is possible. The star occurs in Taylor's Madras Catalogue with the exact date of observation 1833·35. The particulars given are:—No. 5928, B.A.C. 4336, R.A. $12^h\ 46^m\ 50^s{\cdot}70$, Decln. $+8°\ 46'\ 56''{\cdot}53$, magnitude 7, epoch 1835·0. The R.A. was observed once and the Decln. twice. It may be noted that the neighbouring star D.M. $+9°$ 2696 is also given, as of magnitude 7, R.A. $12^h\ 48^m\ 55^s{\cdot}44$, Decln. $+9°\ 11'\ 19''{\cdot}30$, epoch 1835·0, so there can be no question of confusion between the two objects. Taylor's place is not, however, quite in agreement with either Zach or Flamsteed. The R.A. is practically the same as that of Zach, but there is a difference of 4′ in Declination, Taylor's position being the more northerly. Taylor's position is very nearly that of the star $+8°$ 2657, the R.A. being identical, and Taylor's star 1′ to the south.

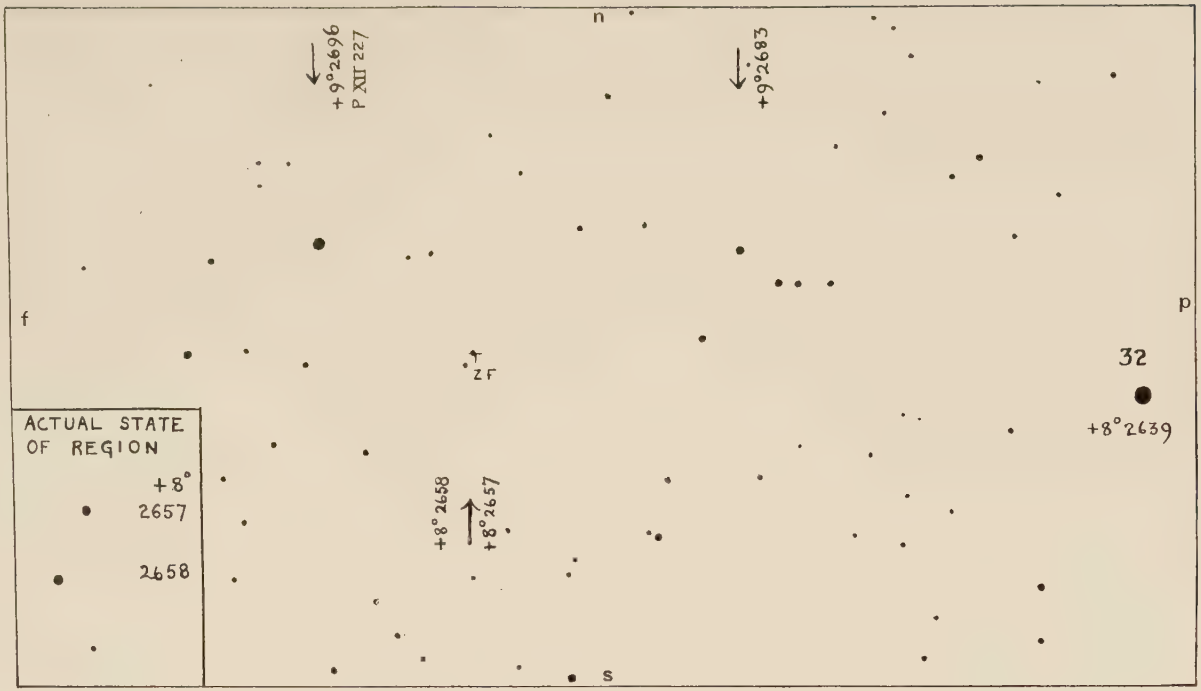
The next and last observation that I have been able to find is by Houzeau, who observed a star, magnitude $6\frac{1}{2}$, that he called 42 Virginis, in 1875·17, on the occasion of the construction of his *Uranometry*. This star was observed in the position of 42, and he included it in the original maps that he made by direct observation of the sky, afterwards identifying it as 42. The only possible explanation of this other than that the real 42 was observed is that Houzeau observed D.M. + 9° 2696 and misidentified it as 42. But this seems out of the question, as the star is well below the 7th magnitude; and although Houzeau observed from equatorial regions, he has included no other stars so faint as this. Furthermore, this star is not included in Gould's *Uranometria Argentina*, which includes stars down to magnitude 7·0, showing that it was fainter than this at the time these observations were made, viz. 1870–4, which is nearly the same as Houzeau's date of observation. The Cordoba observers found magnitude 7·0 as the lowest limit of naked-eye vision under the best conditions. We must therefore discard the idea that Houzeau observed this star and misidentified it as 42, and this leaves it certain that he did observe the true 42.

So far the observations of visibility. As regards observations of non-visibility the first is that of Lalande, who underlines it in a French edition of Flamsteed's Atlas published in 1795. All underlined stars in this work are those which Lalande sought in the sky and could not find. Sir John Herschel did not appear to be aware of this observation of Lalande, which must have been made previous to 1795. In his *Outlines* he gives his own observation of non-visibility:—"I missed it on the 9th of May 1828, and have since repeatedly had its place in the field of view of my 20-feet reflector without perceiving it, unless it be one of two equal stars of the 9th magnitude very nearly in the place it must have occupied." This remark is quite ambiguous as regards evidence whether Sir John had previously actually seen the star or not.

Admiral Smyth says in his *Cycle*, vol. ii. p. 290, that he made a second search for 42 at Baily's request, but only found a 10th magnitude star in the place given in the Astronomical Society's Catalogue, *i.e.* in Zach's position. "This answers to R.A. pretty well, and differs only in Declination by blundering distance; it may therefore be the lost 7 magnitude one retiring." This observation is dated 1838·42. No particulars of his first search are given.

It is singular that one of the most decisive observations of the existence of the star 42, that by Taylor in 1833, should lie between, and relatively close to, Herschel's and Smyth's observations of its non-visibility in 1828 and 1838 respectively.

I have carefully observed the region with a $4\frac{3}{8}$″ refractor several times between December 25, 1913, and the present month, and Herschel's two 9th magnitude stars are readily identifiable. They are D.M. + 8° 2657 and 2658, magnitudes 9·2 and 9·3 respectively.



REGION OF 42 VIRGINIS, PLOTTED FROM THE D.M.

F, T, and Z are positions given by Flamsteed, Taylor, and Zach respectively.
The position of T and the star +8° 2657 are nearly identical.
The position of Z is practically identical with the third star in the small map.

Scale 1 inch = 40 minutes of arc.

Furthermore, Smyth's 10th magnitude is visible, and forms a triangle with these. No other stars can be seen with my instrument near the place of 42.

I have searched the following for the star and it is not contained in any of them:—Bode's *Atlas*, Harding's *Atlas*, Argelander's *Uranometria Nova*, Heis' *Atlas Cœlestis*, Berlin Academy *Sternkarten*, Mayer's *Zodiacal Catalogue*, Rumker's *Mittlere Oerter von* 12,000 *Fix-Sterner*, Santini's *Catalogue of* 1677 *Stars between Equator and* 10° *North Declination*, Lamont's *Munich Zone Catalogue*, *Armagh Catalogue*, *Uranometria Argentina*, *Revised Harvard Photometry;* nor is it in any of the catalogues prior to the time of Flamsteed. The star is clearly marked in Schwink's *Leipzig Atlas* 1843 as 7th magnitude, but I am not aware how far this is an independent authority. It is the only atlas delineating both 42 and +9° 2696 as well as surrounding stars down to the 7th magnitude. The star is found on several modern atlases, doubtless copied from Houzeau.

At my request, Professor E. C. Pickering kindly looked up the region on the Harvard College negatives, and sent me particulars of nineteen photographs of the region. The three stars shown in the neighbourhood of the place of 42 are exactly the same as those I observed visually, and no others are shown. The 10th magnitude star appears constant on ten negatives taken during the period 1891 to 1905, but does not appear on the nine taken between 1906 and 1913, perhaps owing to insufficient exposure, although a more distant star, which Professor Pickering estimates 0·2 magnitude brighter than the above star, is shown on all. This might indicate a slight variability of an irregular kind in this object. Smyth called this star 10th magnitude. According to Knott's table this is equivalent to 9·4 on Argelander's scale. The star is now very markedly fainter than its two neighbours, to which Argelander assigned magnitudes 9·2 and 9·3, the difference being at least a whole magnitude. This must also have been so at the time Argelander observed this region for the D.M., as it is not there included. Neither does he include the star which Pickering estimates 0·2 magnitude greater than Smyth's star. It thus seems probable that Smyth's star is now, and was about 1855, considerably fainter than at the epoch of his observation 1838.

The region of 42 Virginis is outside the track of the planets Uranus and Vesta.

To summarise, there are four certain epochs of observation of a star of the 6–7th magnitude close to the place of 42 Virginis, not counting that of Flamsteed, viz. 1738–45, about 1800, 1833, and 1875. No such star is now visible.

I am indebted to Mr. E. B. Knobel and Mr. W. H. Wesley for kind assistance in the collection of the above data.

Stourwood, Bournemouth:
1914 *June* 8.

The Nebula H.V. 25 Ceti. By Mrs. Isaac Roberts. (Plates 20, 21.)

(*Communicated by the Secretaries.*)

The nebula H.V. 25 Ceti (*h* 56, N.G.C. 246, G.C. 131) was photographed at Starfield, Crowborough, in 1896, and is found, in the Isaac Roberts collection of celestial photographs, on the plates:

			h m		h m		h m	
I.R. 2492, Nov.	9	1896, Exp.	1 30	from	0 20	to	1 50	S.T. Crowborough.
2493, Nov.	10	1896, ,,	2 55	,,	0 33	,,	3 28	,,

and on the 5-inch camera plates I.R. 3406–3407, obtained simultaneously with the 20-inch reflector plates 2492–2493.

The nebula is described as follows in the Isaac Roberts Register of Celestial Photographs:

2492. H.V. 25 Ceti: sky clear; stars round and few; nebula is like the "OWL" M 97, 3 bright stars and 6 very faint condensations involved. H. II. 472 on plate.

2493. H.V. 25 Ceti: sky clear; the description above covers this; nebula a little denser.

The latter negative is reproduced on the scale 1 mm. = 12″ in Dr. Isaac Roberts' volume ii. of *Photographs of Stars, Star-Clusters and Nebulæ*, plate 18, and is described thus on page 123:

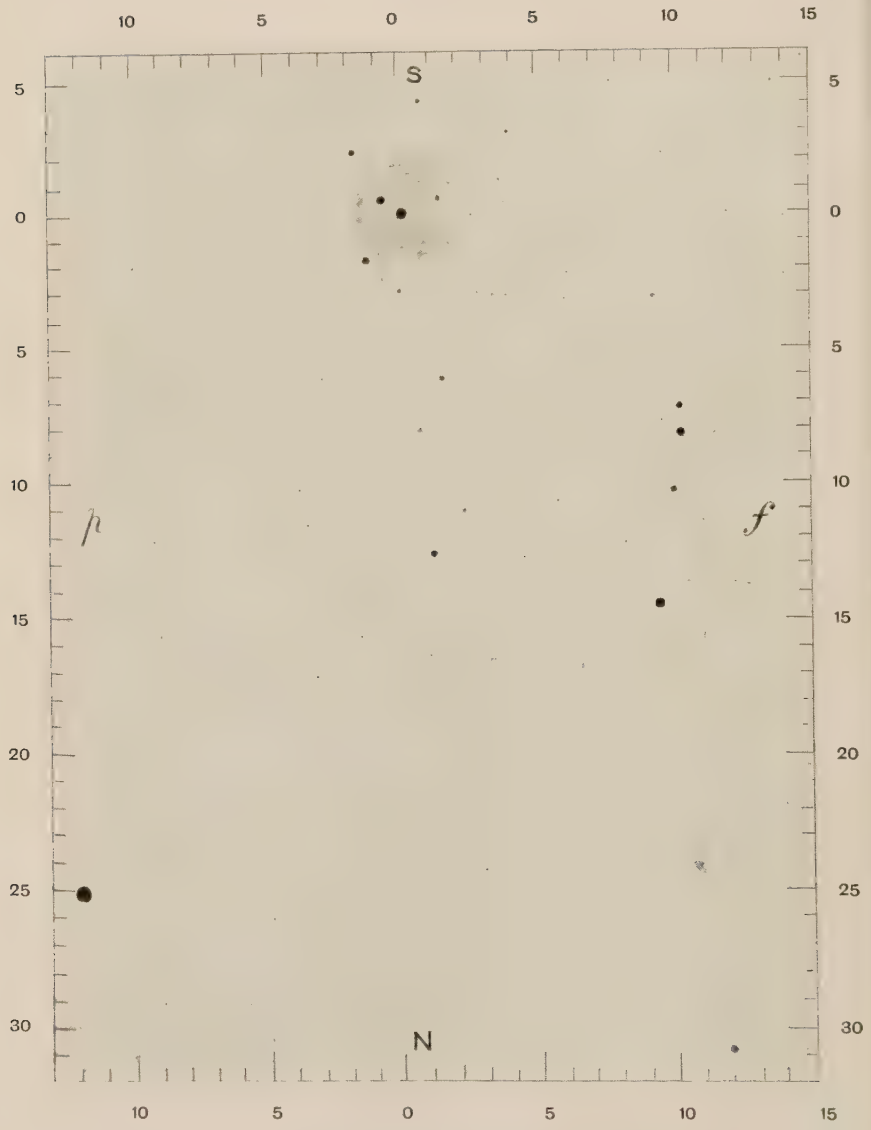
"The photograph shows the nebula to be a very faint spiral with three bright stars apparently involved, and one nearly touching the *np* edge. I think these stars, like others already referred to, are not really involved in the nebula but are seen on projection upon it. There are faint condensations of nebulosity, those near the margin being the most prominent."

Sir John Herschel describes the nebula as "very faint, large, four stars in diffused nebulosity."

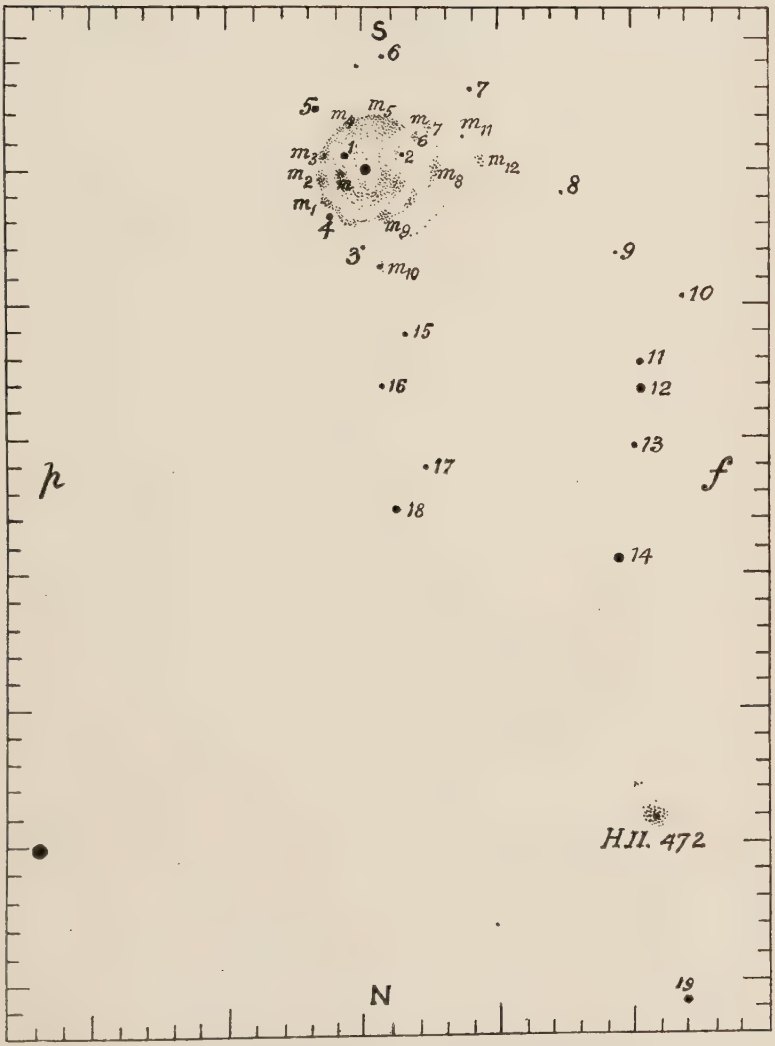
The nebula was too far south for the Birr Castle observers: no description is given thereof by Lord Rosse.

Further Description.—The nebula H.V. 25 Ceti, the largest of the OWL type, is globular, slightly elongated *np* to *sf*, in which direction it covers about 5′ of arc. It is concave towards the centre along its *sp* and *n* margin, where it is limited by nebulous arcs of flocculent nature, of which the masses m_4, m_3, m_2 are the densest. The regularity of the outer curve is broken, in the vicinity of star 4, near mass m_1, where it is concave toward star 4 and convex toward the centre of the nebula; the disturbed arc running over a stretch of about 1′ seems to reveal a physical relation between the star 4 and the nebula.

On its *f* side the nebula is open, formed by nebulosity 19 or



The Nebula H.V. 25 Ceti. I.R. * 2492.


S
N
h
f
H.II. 472
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
m1
m2
m3
m4
m5
m6
m7
m8
m9
m10
m11
m12

KEY MAP.

20 in intensity which gradually blends with the background; there seems to be an outflow of nebulous matter marked by nebulous patches, among which the less faint are m_6, m_7, m_8, m_{11}, m_{12}. On the *f* side of the centre of the nebula is the star 2, which appears almost entirely free from nebulosity along a radius of about 30″; the star seems to repel or absorb the nebulosity.

The star A is near the central region of the nebula; like star 1 it seems to be involved in exceedingly faint nebulosity. To the north of star A is a broad curve of flocculent nebulosity, concave toward A and running almost parallel with the northern marginal curve of H.V. 25. The inner curve turns southward in the vicinity of the large mass of nebulosity which is on the *f* side of m_9; it then runs like a narrow patchy band between the stars A and 2, and expands southward along the southern marginal curve; while fainter and more diffused nebulosity seems to be directed toward the *f* side of the region of bifurcation, thus prolonging the inner spiral (?) curve to the east. A small knot in the inner mass m_0 on the *p* side of star A was measured principally in view of the diagrammatic sketch.

Very faint traces of nebulous curves seem to exist in the outer region of H.V. 25, short arcs of which seem perceptible north of star 3 and south of star 5 and between stars 6 and 7, confirmation of which is desired by a longer exposure.

At a greater distance from the nebula to the *nf* and at the extremity of a stellar arc is the small nebula H. II. 472, which appears globular, with a pretty dense nucleus (intensity 8) involved in nebulosity (intensity 14–19), and which covers about 70″ of arc. Its nucleus and two faint stars in its vicinity were measured. The objects *a*, *β*, *γ*, visible both on I.R. * 2492 and 2493, were also measured, together with stars in their adjacent regions.

In conclusion, the nebula H.V. 25 Ceti seems like a large shell broken along its margin and presenting a wide gap on its *f* side, where there is an outflow of nebulosity. The shell is not hollow, but filled with diffused nebulosity of a flocculent nature which is grouped along certain curves influenced by the presence of the stars A, 1 and 2. The regularity of the marginal curve is disturbed in its *np* region by the presence of star 4, which seems to be in relation with the nebula.

H.V. 25 is remarkable for the absence of well-defined nebulous condensations with dense nebulous nuclei involved in fainter nebulosity, and perhaps may be likened to the great nebula N.G.C. 2237–2239, seen here on a smaller scale and at a later stage of evolution.

Measures.—The nebula H.V. 25 and the stars in its adjacent region were studied on the plate I.R. 2492, upon which the stellar images are better defined than on the plate of $2^h\ 55^m$ exposure. The contrasts in the structure of the nebula are likewise greater on I.R. * 2492 than on I.R. * 2493.

Three series of measures were made by me:

One in the early spring of 1910, with the standard stars A, B,

C, D, F, forming System I., in which all the measures are referred to star A.

A second series in April 1911, with the standard stars B, C, F, I, J, forming System II., in which all measures are referred to the punctured centre of the plate I.R. 2492.

A third series in October 1911, with the standard stars B, C, E, F, G, forming System III., in which all measures are likewise referred to the punctured centre of the plate.*

The constants, which vary with the orientation of the plate in the plate-holder during the exposure to the sky and its orientation on the stage of the Isaac Roberts measuring instrument, were determined for the three series of measures by M. Pourteau of the Paris Observatory.

In System III. the places for 1900·0 of the standard stars B, C, E, F, G were derived from catalogues of visual observations, no photographic positions being available. Neither could available meridian positions be found for the stars A, D, I, J of the *Bonner Durchmusterung*, which were also measured in Series III.

In determining the constants of the plate in System II., it was found necessary to make use of the photographic positions resulting from my measures of Series III.

Likewise, in determining the constants of the plate in the System I., M. Pourteau took into consideration the positions resulting from my measures in the Series III. and II. for the stars A, B, C, and F, which were combined with the positions derived from the catalogues. M. Pourteau also used as standard objects in the System I. the positions of the smaller stars 1, 2, 10, a_0, and H. II. 472 derived from the Systems II. and III.

In the following tables are found the measures of the three series mentioned above: $III_{0.}$, $II_{0.}$, $I_{A.}$ (the measures of III. and II. are referred to the centre of the plate; the measures $I_{A.}$ to star A); the positions resulting from the measures, and the definition, with a brief description of the objects studied on the plate I.R. 2492 are likewise given in the tables annexed. The measures of the faint diffused masses m_0, m_1 . . . m_{12} were extremely difficult. In the Series I. the more prominent knots or dots in the masses were generally measured, whereas in Series II. the whole mass was more particularly considered. The results are comparable.

* The centre of the plate I.R. * 2492 had not been punctured at the time of the first series of measurements.

By Thomery:
1914 *June* 26.

[TABLES.

*Plate I.R. *2492.*

Systems III_0., II_0., and I_A.

Places for 1900·0 of the Standard Stars from which the Constants were derived.

jects.	B.D.			Right Ascension. III.	II.	I.		Declination. III.	II.	I.	Mag.	Authority.
	°	′	h m	s	s	s	°	′	″	″	″	
E	−12	126 K	0 38	47·82	...	..	−12	33	11·8	...	...	6·0 Bossert 2538, Étoiles brillantes.
F	−12	128 Sj	0 39	57·72	57·72	57·67	−12	41	27·1	27·1	27·6	7·8 Gould 673, Santini 41. D.R. III. II.
G	−13	128 K	0 40	27·28	...	...	−13	25	19·8	...	...	6·3 Bossert 91, Mt. propre.
D	−11	135 K	0 40	...	...	31·56	−11	38	...	...	16·0	8·3 D.R. III.
B	−12	132 L	0 41	12·57	12·57	12·50	−11	59	51·6	51·6	52·1	7·3 Paris II. 965. D.R. III. II.
C	−11	139 K	0 41	55·94	55·94	56·00	−11	42	35·4	35·4	34·9	8·0 Paris III. 978. D.R. III. II.
*1	...	...	0 41	...	...	58·38	−12	25	...	...	40·1	... D.R. III. II.
V.25*A	−12	134 ...	0 42	...	...	1·77	−12	25	...	...	7·8	9·9 D.R. III. II.
J	−12	135 ...	0 42	...	4·25	...	−12	40	...	28·4	...	9·5 D.R. III.
*2	...	...	0 42	...	...	7·19	−12	25	...	...	42·7	... D.R. III. II.
$*a_0$	...	...	0 42	...	...	22·43	−11	52	...	...	28·8	... D.R. III. II.
.II. 472	...	...	0 42	...	...	45·55	−12	0	...	...	55·5	... D.R. III. II.
*10	...	...	0 42	...	...	50·10	−12	20	...	...	18·5	... D.R. II.
I	−12	143 ...	0 44	...	35·41	...	−12	45	...	1·5	...	9·0 D.R. III.

Measures of Rectangular Co-ordinates.

jects.	III_0. x	y	II_0. x	y	I_A. x	y	III_0. Diam.		II_0. Diam.		I_A. Diam.		Int.
	′	′	′	′	′	′	″	″	″	″	″	″	
E	−41·927	− 2·663	...	...	...	...	41	38	...	...	...	...	2
F	−25·043	−11·083	−24 955	−11·301	−29·609	−17·560	36	32	34	32	33	31	2
G	−18·344	−54·876	...	...	...	...	47	45	...	...	...	...	2
D	−15·956	+51·865	...	...	−23·851	+45·797	34	34	...	...	34	33	2
B	− 6·240	+30·211	− 6·513	+30·161	−13·022	+24·665	30	31	32	31	31	30	1
c. pl.	0·000	0·000	0·000	0·000	...	...	...	...	...	...	...	...	...
C	+ 4·660	+47·350	+ 4·244	+47·376	− 3·028	+42·348	31	29	33	31	31	29	2
*1	+ 4·619	+ 4·372	+ 4·551	+ 4·401	− 0·806	− 0·568	19	17	18	15	18	16	5
V.25*A	+ 5·442	+ 4·898	+ 5·394	+ 4·936	...	...	22	20	20	19	21	21	3
J	+ 5·840	−10·408	+ 5·909	−10·355	...	...	28	19	19	19	...	...	4, 14
*2	+ 6·762	+ 4·301	+ 6·705	+ 4·354	+ 1·348	− 0·532	10	9	12	10	11	12	6, 9
$*a_0$	+10·957	+37·397	+10·644	+37·485	+ 3·828	+32·746	16	16	18	17	17	16	6, 14
.II. 472	+16·479	+28·916	+16·244	+29·041	+ 9·746	+24·565	9	9	10	9	9	9	15, 18
*10	...	...	+17·212	+ 9·712	+11·622	+ 5·314	...	...	9	7	10	6	11, 19
I	+42·601	−15·416	+42·724	−15·057	...	...	30	29	29	28	...	...	2

*The Constants and Concluded Centre of the Plate I.R. *2492.*

System III_0.		System II_0.	
$t_x = +0\cdot00078$	$t_y = +0\cdot00278$	$t_x = +0\cdot00045$	$ty = +0\cdot00256$
$i_x = -0\cdot01389$	$i_y = -0\cdot01117$	$ix = -0\cdot00584$	$i_y = -0\cdot00221$
$\alpha_0 = 0^h\ 41^m\ 39^s\cdot70$	$\delta_0 = -12°\ 30'\ 6''\cdot1$	$\alpha_0 = 0^h\ 41^m\ 39^s\cdot83$	$\delta_0 = -12°\ 30'\ 5''\cdot5$

Constants referred to Star A, and Concluded Positions thereof.

System I_A.

$t_x = -0\cdot00062$	$t_y = +0\cdot00167$
$i_x = +0\cdot03829$	$iy = +0\cdot04163$
$\alpha_0 = 0^h\ 42^m\ 1^s\cdot76$	$\delta_0 = -12°\ 25'\ 7''\cdot9$

Concluded Positions for 1900·0 *of the Standard Stars in the Systems III_0., II_0., and I_A.*

Object.	Right Ascension.				Declination.			
		III_0.	II_0.	I_A.		III_0.	II_0.	I_A.
	h m	s	s	s	° ′	″	″	″
E	0 38	47·91	...	...	−12 33	10·9	...	...
F	0 39	57·57	57·74	57·68	−12 41	28·5	27·4	27·5
G	0 40	27·35	...	...	−13 25	19·2	...	...
D	0 40	31·56	...	31·58	−11 38	16·0	...	15·2
B	0 41	12·45	12·47	12·41	−11 59	52·6	52·0	52·8
c. pl.	...	...	...	...	...	...	...	...
C	0 41	56·06	56·00	56·02	−11 42	34·2	35·2	35·3
*1	0 41	58·39	58·37	58·37	−12 25	39·9	40·2	40·0
H.V. 25 *A	0 42	1·73	1·81	...	−12 25	7·7	7·8	...
J	0 42	4·25	4·31	...	−12 40	28·4	27·4	...
*2	0 42	7·17	7·20	7·19	−12 25	42·7	42·6	43·2
$*\alpha_0$	0 42	22·40	22·46	22·52	−11 52	28·6	29·1	29·5
H. II. 472	0 42	45·50	45·59	45·44	−12 0	54·9	56·0	55·7
*10	0 42	...	50·10	50·15	−12 20	...	18·5	17·2
I	0 44	35·41	35·47	...	−12 45	1·5	2·0	...

The Nebula H.V. 25 Ceti (in the Systems II_0. and I_A.)

Object.	Measured Co-ordinates.				Concluded Places for 1900·0.					Diameters.				Intensity.	
					Right Ascension.		Declination.								
	II_0.		I_A.		II_0.	I_A.	II_0.	I_A.		II_0.		I_A.		II_0.	I_A.
	x	y	x	y											
					h m	s	s	° ′	″	″	″	″	″		
m_2	+ 3′·781	+ 5′·260	− 1′·646	+ 0′·081	0 41	55·20	55·04	− 12 24	48·6	58·9	33	18	8	7	16	17
m_3	+ 3′·828	+ 4′·341	− 1′·576	− 0′·489	0 41	55·41	55·23	− 12 25	43·9	33·3	35	22	9	7	15, 16	16
,,	...	...	− 1′·662	− 0′·454	0 41	...	54·89	− 12 25	...	31·1	...	...	12	8	...	15
m_1	+ 4′·263	+ 6′·258	− 1′·051	+ 1′·369	0 41	57·15	57·67	− 12 23	48·5	43·0	28	23	14	10	17	16
m_0	...	...	− 0′·708	+ 0′·663	0 41	...	58·97	− 12 24	...	26·3	...	...	8	7	...	16
m_4	+ 5′·276	+ 3′·273	+ 0′·019	− 1′·646	0 42	1·37	1·58	− 12 26	47·8	46·8	30	18	6	4	16	17
,,	...	...	− 0′·271	− 1′·616	0 42	...	0·40	− 12 26	...	44·3	...	...	8	6	...	16
m_9	+ 6′·101	+ 6 501	+ 0′·681	+ 1′·625	0 42	4·67	4·80	− 12 23	33 6	32·0	29	19	28	19	16, 19	16, 18
m_5	+ 6′·453	+ 3′·279	...	...	0 42	6·20	...	− 12 26	47·3	...	46	46	...	...	17	...
m_5	+ 6′·461	+ 3′·386	+ 1′·046	− 1′·391	0 42	6·23	5·82	− 12 26	40·9	34·1	52	29	11	11	17	16
,,	...	...	...	...	0 42	6·21	...	− 12 26	44·1	...	...	...	...	...	...	...
m_6	...	...	+ 1′·873	− 0′·915	0 42	...	9·28	− 12 26	...	7·6	...	...	9	9	...	15, 16
,,	...	...	+ 1′·902	− 0′·991	0 42	...	9·39	− 12 26	...	12·2	...	...	10	7	...	17
,,	+ 7′·227	+ 3′·927	...	...	0 42	9·35	9·33	− 12 26	8·2	9·9	33	16	...	...	17, 18	...
m	...	...	+ 2′·423	− 1′·115	0 42	...	11·50	− 12 26	...	21·0	...	...	14	14	...	17
m_8	+ 7′·772	+ 4′·611	...	...	0 42	11·56	...	− 12 25	27·0	...	34	23	...	...	18	...
m_{10}	...	...	+ 0′·365	+ 3′·829	0 42	...	3·85	− 12 21	...	18·7	...	...	11	10	...	17
m_{11}	...	...	+ 3′·571	− 1′·192	0 42	...	16·19	− 12 26	...	28·5	...	...	6	5	...	17
m_{12}	+ 9′·647	+ 4′·533	+ 4′·285	− 0′·556	0 42	19·25	19·21	− 12 25	31·4	52·0	18	14	8	5	17	17

The measured co-ordinates and the concluded places for the stars A, 1 and 2 involved in the nebula H. V. 25 are given in tables, pp. 715, 716.

Small Stars in the Region

Objects.	Measured Co-ordinates. III$_0$. x	III$_0$. y	II$_0$. x	II$_0$. y	I$_A$. x	I$_A$. y	Diameters. III.	III.	II.	II.	I.	I.
*5	+3′·605	+2′·724	+ 3′·571	+ 2′·752	− 1′·732	− 2′·275	15″	13″	16″	14″	15″	14″
*4	+4·144	+6·614	+ 4·058	+ 6·644	− 1·402	+ 1·633	15	15	15	15	16	14
*3	+5·350	+7·747	+ 5·265	+ 7·786	− 0·252	+ 2·843	8	7	9	8	8	7
,,	...	...	...	...	− 0·186	+ 2·831	...	...	...	...	9	6
6	+6·091	+0·881	+ 6·076	+ 0·922	...	...	14	12	15	13	...	...
16	...	...	+ 6·110	+13·017	...	...	...	...	8	7	...	...
18	...	...	+ 6·516	+17·446	...	...	...	...	16	15	...	...
15	...	...	+ 6·942	+11·165	+ 1·371	+ 6·251	...	...	7	6	10	7
17	...	...	+ 7·570	+15·948	...	...	...	...	10	9	...	...
7	+9·347	+1·949	+ 9·298	+ 2·014	...	...	9	8	10	9	...	...
8	...	...	+12·617	+ 5·817	+ 7·151	+ 1·038	...	...	13	9	9	9
,,	...	...	+12·620	+ 5·829	+ 7·268	+ 1·070	...	...	13	9	13	9
9	...	...	+14·641	+ 8·164	+ 9·161	+ 3·518	...	...	7	5	10	10
,,	...	...	+14·646	+ 8·156	...	...	...	...	6	4	...	...
14	...	...	+14·913	+19·444	...	...	...	...	20	20	...	...
13	...	...	+15·426	+15·159	+ 9·690	+10·545	...	...	16	14	16	15
11	...	...	+15·585	+12·100	+ 9·982	+ 7·491	...	...	16	13	17	15
12	...	...	+15·703	+13·071	+10·068	+ 8·470	...	...	18	17	20	17
19	...	...	+17·364	+35·684	...	...	...	...	15	12	...	...

Other Faint Nebulæ on I.R. *2492, *and*

Object.	Measured Co-ordinates. III$_0$. x	III$_0$. y	II$_0$. x	II$_0$. y	I. x	I. y	Diameters. III.	III.	II.	II.	I.	I.
H.II.472	+16′·479	+28′·916	+16′·244	+29′·041	+ 9′·746	+24′·565	9″	9″	10″	9″	9″	9″
,,	+16·471	+28·917	...	...	...	..	...	...	...	...	...	...
*b_1	+15·724	+27·646	+15·507	+27·787	+ 9·053	+23·256	7	7	8	7	8	7
*b_2	+19·126	+27·291	...	...	...	...	11	11	...	...	...	...
,,	+19·126	+27·288	...	...	..	...	...	...	...	..	...	...
a	+11·136	+39·711	+10·746	+39·824	+ 3·777	+35·137	14	13	22	19	19	15
,,	...	...	...	...	+ 3·790	+35·112	.	...	...	..	...	...
*a_1	...	...	...	...	+ 3·540	+33·902	...	...	...	...	11	10
*a_2	...	...	...	...	+ 2·886	+35·956	...	...	...	...	7	5
*a_3	...	...	...	...	+ 3·195	+36·279	...	...	...	...	10	10
*a_4	+13·592	+31·613	...	...	...	...	9	8	...	...	..	..
β	− 9·218	+26·751	− 9·465	+26 686	−15·802	+21·069	9	9	12	10	9	9
γ	+30·251	+10·901	+30·154	+11·151	...	...	5	5	5	5	..	..
,,	+30·248	+10·903	...	...	...	...	...	...	...	...	...	..
*κ	+33·296	+ 8·097	...	...	...	..	11	10	...	..	...	..

of H.V. 25 Ceti.

Objects.	Concluded Positions for 1900·0. Right Ascension. III_0.	II_0.	I_A.	Declination. III_0.	II_0.	I_A.	Intensity. III.	II.	I.
	h m s	s	s	° ′	″	″			
*5	0 41 54·32	54·40	54·31	−12 27	19·8	19·5	20·3	7	4	...
4	0 41 56·31	56·30	56·28	−12 23	25·4	25·3	26·3	5	3	...
3	0 42 1·18	1·21	1·18	−12 22	16·2	16·4	16·4	12	12	...
,,	0 42 ...	...	1·44	−12 22	...	...	17·3	...	...	15
6	0 42 4·62	4·71	...	−12 29	8·9	9·2	...	7	6	...
16	0 42 ...	4·54	...	−12 17	...	1·7	...	...	15	...
18	0 42 ...	6·09	...	−12 12	...	35·1	...	...	4	...
15	0 42 ...	8·00	8·35	−12 18	...	52·9	55·7	...	13	...
17	0 42 ...	10·44	...	−12 14	...	5·0	...	...	14	...
7	0 42 17·91	17·89	...	−12 28	2·4	3·0	...	9	8, 11	...
8	0 42 ...	31·39	31·19	−12 24	...	13·6	23·3	...	17	17
,,	0 42 ...	31·40	31·68	−12 24	...	12·9	21·7	...	...	15
9	0 42 ..	39·62	39·80	−12 21	...	52·1	59·1	...	9	11
,,	0 42 ...	39·64	...	−12 21	...	52·6	...	...	9, 16	...
14	0 42 ...	40·42	...	−12 10	...	33·5	...	...	3	...
13	0 42 ...	42·63	43·05	−12 14	...	51·1	58 1	...	5	5
11	0 42 ..	43·37	43·79	−12 17	...	55·1	62·4	...	5	4
12	0 42 ...	43·83	44·28	−12 16	...	56·6	63·8	...	3, 4	2
19	0 42 ...	49·99	...	−11 54	...	16·2	...	...	6	...

Stars in the Adjacent Regions.

Object.	Concluded Positions for 1900·0 in the Systems III_0., II_0., and I_A. Right Ascension. III_0.	II_0.	I_A.	Declination. III_0.	II_0.	I_A.	Intensity. III.	II.	I.
	h m s	s	s	° ′ ″	″	″			
H.II.472	0 42 45·50	45·59	45·44	−12 0 54·9	56·0	55·7	15, 18	8, 14, 19	15, 18
,,	0 42 45·47	...	...	−12 0 54·8	...	...	...	...	...
*b_1	0 42 42·49	42·62	42·41	−12 2 11·7	11·6	12·6	17	17	16
*b_2	0 42 56·43	...	...	−12 2 30·6	...	...	10	...	...
,,	0 42 56·43	...	...	−12 2 30·8	...	...	...	...	...
a	0 42 22·99	22·81	22·68	−11 50 9·3	8·5	5·7	18, 19, 20	17, 19	18, 20
,,	0 42 ...	...	22·73	−11 50 ...	...	7·2	...	...	...
*a_1	0 42 ...	...	21·52	−11 51 ...	...	19·3	...	...	13
*a_2	0 42 ...	...	19·17	−11 49 ...	...	14·2	...	...	17
*a_3	0 42 ...	...	20·48	−11 48 ...	...	55·5	...	...	11
*a_4	0 42 33·52	...	...	−11 58 14·7	...	...	9	...	...
β	0 41 0·45	0·46	0·47	−12 3 22·6	21·4	21·7	14	...	10, 20
γ	0 43 43·03	43·07	...	−12 18 48·3	49·0	...	14, 18, 19	15, 16, 17	...
,,	0 43 43·02	...	...	−12 18 48·1	...	...	...	...	...
κ	0 43 55·69	...	...	−12 21 34·5	...	...	9	...	...

DEFINITION AND DESCRIPTION IN THE SYSTEMS (II.) AND (I.).

A, bright star involved in H.V. 25, seems to be surrounded by very faint nebulosity.

1, star in the *p* region of H.V. 25, involved in extremely faint nebulosity.

2, in the *f* region of H.V. 25; seems free from nebulosity.

m_0, very faint knot in the inner spiral (?) branch (I.).

m_1, nebulous mass in a streak of nebulosity south of stars 4 (II.); densest mass measured (I.).

m_2, broken-up mass on *p* margin (II.).
Faint knot between two fainter ones measured (I.).

m_3, nebulous streak at *p* border—disaggregated (II.).
Faint knot in the nebulous streak measured (I.).

m_4, nebulous streak at *s* margin (II.).
Faint knot in nebulous band measured (I.).
Faint knot at the *f* extremity of nebulous band measured (I.).

m_5, large diffused mass measured (II.).
Faint knot measured (I.).
Mean values of m_5 in System (II.) inscribed.

m_6, large nebulous mass measured (II.).
Nebulous knot measured (I.), another knot measured (I.); mean values inscribed, which agree with (II.).

m_7, extremely faint nebulous mass near m_6 (I.).

m_8, nebulous mass in *f* region of H.V. 25 (II.).

m_9, large diffused oval mass near *n* margin (II.) (I.).

m_{10}, small diffused mass in exterior region (I.).

m_{11}, extremely faint nebulous knot in *sf* region of H.V. 25 (I.), other similar knots near limit of visibility.

m_{12}, extremely faint star or nebulous knot measured (II.) (I.).

3, faint star with traces of nebulosity (?) in its *n* region measured twice (I.).

4, star in concave arc of small curve.

5, 6, 7, stars in the southern region of H.V. 25, with traces of nebulosity (?) in their vicinity.

8, very faint star, irregular, measured twice (II.) (I.).

9, 10, stars with diffused outlines, *9 measured twice (II.).

11, 12, 13, 14, small faint stars in a curve leading to H. II. 472.

15, 16, 17, 18, faint stars north of H.V. 25.

19, pretty dense star north of H. II. 472.

H. II. 472, small faint globular (?) nebular intensity 15–18, covering about 70″ and presenting a nucleus intensity 8.

b_1, b_2, faint stars near H. II. 472; b_2 measured twice (III.).

a, extremely faint nebulous nucleus with nebulous rays directed *sf* to *np*; also on I.R.* 2493; is perhaps a small nebula seen edgewise; measured twice (I.).

a_0, faint star, intensity 6, with diffused margin, used as standard star in System I.

$a_{1,2,3}$, small stars in the region of *a*.

β, star-like object with very faint nebulosity (?) in its *n* region (a longer exposure is desired); is *sp* star B and also on I.R.* 2493.

γ, faint nebulous nucleus with short arcs of nebulosity in its *nf* region; also on I.R. * 2493 with similar aspect, measured twice (III.).

K, star intensity 9 *sf* γ.

Maison de Rosa Bonheur, By Thomery:
1914 *May* 29.

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(1) *Dimensions of Saturn and his Rings as measured on Professor Barnard's Photograph of* 1911 *November* 19; (2) *Observations on the Transparency of Ring A, and other Details appearing on the Photograph.* By P. H. Hepburn. (Plate 22.)

This paper deals with measures of Saturn and the ring-system made on the photograph of the planet taken by Professor Barnard with the 60-inch reflector at Mount Wilson on 1911 November 19, and with certain features of ring A observable thereon. A glass positive of this photograph is in the possession of the Society. There are twelve exposures on the plate, which is thus described in the *Annual Report* of the Mount Wilson Observatory for 1912, page 197:—

"The original negative shows the belt, polar caps, and rings very well defined. The Cassini division is strongly shown, and feeble traces of the Encke division appear. The exposures were ordinarily from 10 to 12 seconds. One exposure of 40 seconds shows the crape ring fairly well, though the rest of the planet is, of course, over-exposed in this case."

The equivalent focal length of the telescope was 100 feet, and an enlarging lens and yellow colour screen were used. The images measure about $13\frac{1}{2}$ millimetres along the major axis of the ring.

In the print in the possession of the Society, with few exceptions all outlines are beautifully sharp, and the image stands high magnification. It is doubtful whether any trace of the crape ring in the ansæ is visible, and Encke's division is represented by a fairly well-defined difference of shade between the inner and outer portions of ring A.

The following data are given in the *Nautical Almanac* for the date in question:—

Diff. heliocentric longitude, earth and planet	8° 58′·6
Elevation of earth above plane of ring .	21° 3′·4 (S)
„ sun „ „ .	21° 36′·4 (S)
Date of opposition	Nov. 9 18^h

The photograph was therefore taken about ten days after opposition. The globe casts a narrow shadow on the ring on the following side. A narrow margin (about 0″·15 in breadth) of the shadow of ring A on the ball might be expected to be visible beyond the north edge of this ring, but there is no trace of this except doubtfully on image 8. Phase is inappreciable.

The writer's attention was some time since directed to the suitability of this photograph for accurate measurement.

In March 1914 a note appeared in *l'Astronomie*, p. 146, giving measures made by Herr Graff of Bergedorf Observatory, Hamburg, on a process-block illustration of this print. The writer compared the figures with the measures made by Barnard in 1894–5, and found very considerable discrepancies. He therefore took steps to obtain accurate measures of the Society's transparency. This work was very kindly carried out by Mr Hollis of the Royal Observatory, to whom the writer desires to express his great indebtedness, and also to the Society for giving the necessary facilities. A further set of measures has been made by the writer, by the method described below, and both sets are dealt with in this paper. In the course of this investigation the writer observed a remarkable fact, which he believes to have been hitherto unrecorded, that *the outer ring of Saturn, as shown on this photograph, is transparent.*

Mr. Hollis's measures were made on the glass positive, and are described by him as follows:—

"These measures have been made with the Greenwich solar micrometer, low-power eyepiece. Bisections or settings were made by setting a fiducial spot in the eyepiece on the appropriate point of the photograph and making a reading of the vernier on the scale. The scale is divided to ·01 of 4 inches, and the vernier reads to ·001 of 4 inches, so that the unit of measurement is of the order of a tenth of a millimetre. The figures give distances in this unit from an adopted centre found by taking a mean of all the readings for east and west directions, and of north and south pole readings for north and south. The Cassini division is generally seen as a line only, and the centre of this is measured. All the figures in the table are the mean of two settings."

The images measured were Nos. 2, 4, 5, 6, 9, and 10.

The scale readings were reduced by the writer to seconds by equating the external diameter of ring A derived from the mean of the above (excluding 10) to the diameter given by Barnard's measures 1894–5, namely, 40″·186.

As so reduced they are now set out in Table I.

TABLE I.

No. of Image.	Ring A Exterior. E.–W.	Ring A Exterior. N.–S.	Ring A Interior. E.–W.	Centre Cass. Div. N.–S.	Ring B Exterior. E.–W.	Ring B Interior. E.–W.	Ring B Interior. N.–S.	Ring C Interior. N.–S.	Equatorial Diameter. E–W.	Polar Diameter. N.–S.
	″	″	″	″	″	″	″	″	″	″
10	39·97	38·98	35·46	...	34·74	26·19	25·42	...	17·97	16·29
2	39·90	38·98	35·72	33·92	34·87	26·90	25·84	23·34	17·41	15·83
4	40·67	39·48	35·50	33·84	34·85	26·68	26·34	23·50	17·92	15·61
5	40·15	40·20	35·94	36·92	34·64	26·73	27·64	23·84	17·65	15·99
6	40·30	39·42	35·78	34·56	34·83	26·63	26·28	23·14	17·71	16·11
9	39·90	40·34	35·65	...	34·58	26·72	26·84	...	17·47	16·03
Mean (2–9)	40·19	39·68	35·72	34·81	34·75	26·73	26·59	23·45	17·63	15·91
±	·11	·22	·05	1·81	·05	·03	·25	·29	·07	·07

The results of these measures (as well as of those next described) can, of course, only be taken as giving the proportions of the various members of the system, not the absolute dimensions.

A second set of measures was made by the writer as follows:—

A negative taken from the original transparency was projected by means of an ordinary optical lantern on to a rigid vertical screen, care being taken to get screen and lantern exactly square. Each of the twelve images was projected in succession on to the same small area in the centre of the field. The distance from the screen was so regulated that the extreme diameter of ring A measured 201 millimetres, this scale being chosen because it permitted a convenient comparison with Barnard's measures of 1894–5, which give 20″·093 for the exterior semi-diameter of this ring. The scale of the measures may therefore be taken as 5 millimetres to the second very nearly. The figures so obtained are doubled for statement in this paper. Each image was projected on to a separate sheet of paper ruled with two rectangular axes intersecting near the middle of the sheet, at a point which will be called the adopted centre. On the horizontal axes dots were marked at such a distance from the adopted centre as to come about 2 millimetres outside the outer extremities of ring A. The image having been centred on the screen, the paper was pinned in position, so that the horizontal axis approximated as closely as could be judged by the eye with the major axis of the rings, and the two dots were approximately the same distance outside the extremities of ring A. Owing to the dissimilarity between the polar limbs, it was not considered advisable to attempt to centre the image north and south by similar means, but reliance was placed on the equal division by eye of the four quadrants into which the space between the inner edge of ring B and the planet was divided by the horizontal axis and the limb of the ball. The intersections of the four axes by the edges of the rings were then carefully plotted in pencil, and the distance of each of these points from the adopted centre was subsequently measured. Thus, so

far as the measures along the major axis are concerned, any error of centering should be eliminated, and a re-examination of the cases in which marked discrepancies occurred seems to show that they are in no case due to defective centering.

The south limb is so diffuse on most of the images as to be very unreliable. One could only trust that errors would be equalised by the independent measures of the twelve images. The whole of the plotting was carried out at one sitting by an expert draughtsman, and it is apprehended that in few cases does the error due to inaccurate plotting exceed 0·2 millimetre. With the exception of the outer edge of ring A, and N. and S. limbs of the ball, the edges measured show, in nearly all cases, a penumbra of the mean width of about 1·2 millimetres. Both edges of these penumbræ were plotted, no attempt being made to average them.

After all the diagrams had been plotted, they were measured with a scale divided in millimetres, and estimated by eye to tenths of a millimetre. Two sets of measures were made of images 8 and 11. In taking the final mean the measures of image 10 were excluded, and images 8 and 11 were represented by the means of the two sets, so that each of the included 11 images is treated as carrying equal weight. The detailed measures of the outer (or faint) edges are set out in Table II. The figures are those of the actual measures multiplied by 2, and contain the constant correction about to be referred to.

As stated above, the images were centred to the outer extremities of ring A. The following table, comparing the radial measures (preceding and following) of all the limbs of the ring-system, indicates that the exterior edge of ring A is eccentric.

	Mean.	Preceding.	Following.	Residual M − P.
1. Ring A extr.	20·102	20·128	20·076	− ·026
2. ,, intr. (dense edges)	18·328	18·280	18·376	+ ·048
3. ,, ,, (faint edges)	18·100	18·058	18·142	+ ·042
4. Ring B extr. ,,	17·426	17·398	17·454	+ ·028
5. ,, ,, (dense edges)	17·270	17·260	17·280	+ ·010
6. ,, intr. ,,	13·858	13·812	13·904	+ ·046
7. ,, ,, (faint edges)	13·570	13·524	13·616	+ ·046
Mean residual (2) to (7)	...	...	...	+ ·037

A constant correction of 0·04 was therefore added to all the measures on the P side, and subtracted from those on the F side, equivalent to shifting the adopted centre 0·04 millimetre in the latter direction.

The excessive width of ring A on the preceding side is appreciable on inspection of the enlarged photograph. With this exception no eccentricity in the ring-system is indicated.

The north and south measures, owing to the diffuse character of the images of the south limb, are subject to much greater uncertainty than those made east and west. The mean of the

TABLE II.

Measures of Barnard's Photograph, 1911 Nov. 19 (Hepburn).

(*Faint Edges of Penumbræ of Rings and Ball.*)

Image.	Outer Radius of Ring A.			Inner Radius of Ring A.			Outer Radius of Ring B.			Inner Radius of Ring B.			Inner Radius of Ring C.	Equatorial Radius.		Polar Semi-diameter of Disc.	
	Pre-ceding.	Follow-ing.	From N. and S. Measures.	Pre-ceding.	Follow-ing.	From N. and S. Measures.	Pre-ceding.	Follow-ing.	From N. and S. Measures.	Pre-ceding.	Follow-ing.	From N. and S. Measures.	From N. and S. Measures.	Pre-ceding.	Follow-ing.	N.	S.
10	20·44	20·22	21·04	18·14	17·86	...	17·82	17·50	...	13·32	12·92	...	...	9·16	8·86	8·20	8·12
1	20·24	20·16	20·04	18·20	18·30	...	17·36	17·40	16·92	13·70	13·68	13·20	11·20	8·72	8·86	8·08	8·00
2	20·22	20·02	20·26	18·22	18·02	...	17·46	17·20	...	13·52	13·40	13·02	10·92	8·72	8·88	8·00	8·04
3	20·18	20·04	19·66	18·04	18·28	18·14	17·40	17·60	17·26	13·52	13·72	13·36	10·96	8·66	8·78	8·06	7·80
4	20·18	20·12	...	18·08	18·00	...	17·48	17·58	...	13·42	13·62	13·52	11·46	8·74	9·00	8·26	7·80
5	20·02	19·96	...	17·84	17·64	...	17·20	17·34	...	13·46	13·62	13·86	11·14	8·60	8·88	7·88	7·88
6	20·30	19·94	20·14	18·08	18·10	...	...	...	...	13·40	13·60	13·36	10·92	8·80	8·72	7·96	7·96
7	20·18	20·16	20·14	...	...	17·82	17·58	17·30	17·20	13·66	13·40	13·26	11·64	8·84	8·80	8·00	7·86
8	20·28	20·03	19·90	18·26	18·09	18·18	17·59	17·28	17·18	13·70	13·60	13·86	11·42	8·91	8·69	7·93	7·91
9	20·14	20·02	19·94	...	...	18·38	17·40	17·40	17·42	13·64	13·50	...	11·46	8·82	8·64	7·98	7·92
11	20·11	19·84	19·98	18·17	18·06	...	17·50	17·42	16·86	13·80	13·44	13·70	11·52	8·83	8·57	7·88	7·77
12	20·00	20·10	20·48	18·00	18·42	18·60	17·40	17·62	17·70	13·38	13·76	13·30	11·42	8·54	8·88	8·03	7·88
Mean (excl. 10)	20·168	20·036	20·060	18·099	18·101	18·224	17·437	17·414	17·220	13·564	13·576	13·444	11·278	8·744	8·791	8·010	7·90
±	·002	·002	·105	·030	·051	·175	·024	·031	·146	·029	·026	·120	·106	·023	·026	·044	·035

measures from the adopted centre to the south limb is 7″·90, as against 8″·01 to the north limb. It was assumed that errors of centering cancel out, and the whole difference was attributed to defect of the image of the south limb. The adopted centre was thus left undisturbed north or south. The north and south measures of the rings are affected by this assumption, but it seems to be borne out to some extent by the close accord of the dimensions of the outer members of the ring-system reduced therefrom with those measured directly on the horizontal axis.

The displacement of the adopted centre horizontally involves a small correction to the north and south measures, but since it is less than 0″·001 for the radius, it has been neglected.

The polar diameter of the globe deduced from the mean north and south radius of 8″·01, and the mean equatorial radius (measured to the outer edge of penumbra) of 8″·767, is 15″·78. But owing to the uncertainty stated above no great reliance can be placed on these figures.

The following table gives the best measures of recent years. The quantities in the first four columns are highly concordant, and their mean may apparently be taken as representing the visual dimensions of Saturn during the period 1885–1900. See's results of 1901 are in the main the 1900 measures corrected for irradiation by daylight measures made during that year.

TABLE III.

	1. Hall, 1885–7.	2. Barnard, 1894–5.	3. Lewis and Dyson, 1895.	4. See, 1900.	5. Mean of 1–4.	6. See, 1901.
	″	″	″	″	″	″
Ring A extr.	40·450	40·186	40·590	40·274	40·375	39·971
,, intr.	34·950	35·034	34·870	34·757	34·902	34·605
Centre C. Div.	34·530	34·517	34·349	34·339	34·434	34·139
Ring B extr.	34·110	34·000	33·828	33 921	33·965	33·671
,, intr.	25·750	25·626	25·647	25·932	25·739	25·932
Ring C intr.	20·520	20·528	20·765	20·434	20·562	20·434
Saturn, equat. diameter	17·720	17·798	17·754	17·804	17·769	17·240
Saturn, polar diameter	...	16·246	16·793	[16·005]	16·348	[15·494]
Width, ring A	2·750	2·576	2·860	2·758	2·736	2·455
,, C. Div.	0·420	0·517	0·521	0·418	0·469	0·818
,, ring B	4·180	4·187	4·091	3·995	4·113	3·745
,, ring C	2·625	2·549	2·382	2·749	2·576	2·749

Notes to Table III.—

Columns 1 and 2 from *M.N.*, vol. lvi. p. 14.

Column 3 from *M.N.*, vol. lvi. p. 14, as subsequently corrected by Professor Barnard.

Columns 4 and 6 from *Ast. Nach.*, February 1902. The figures are given as there stated, but appear to contain errors. Polar diameters not measured but calculated from H. Struve's value for ellipticity.

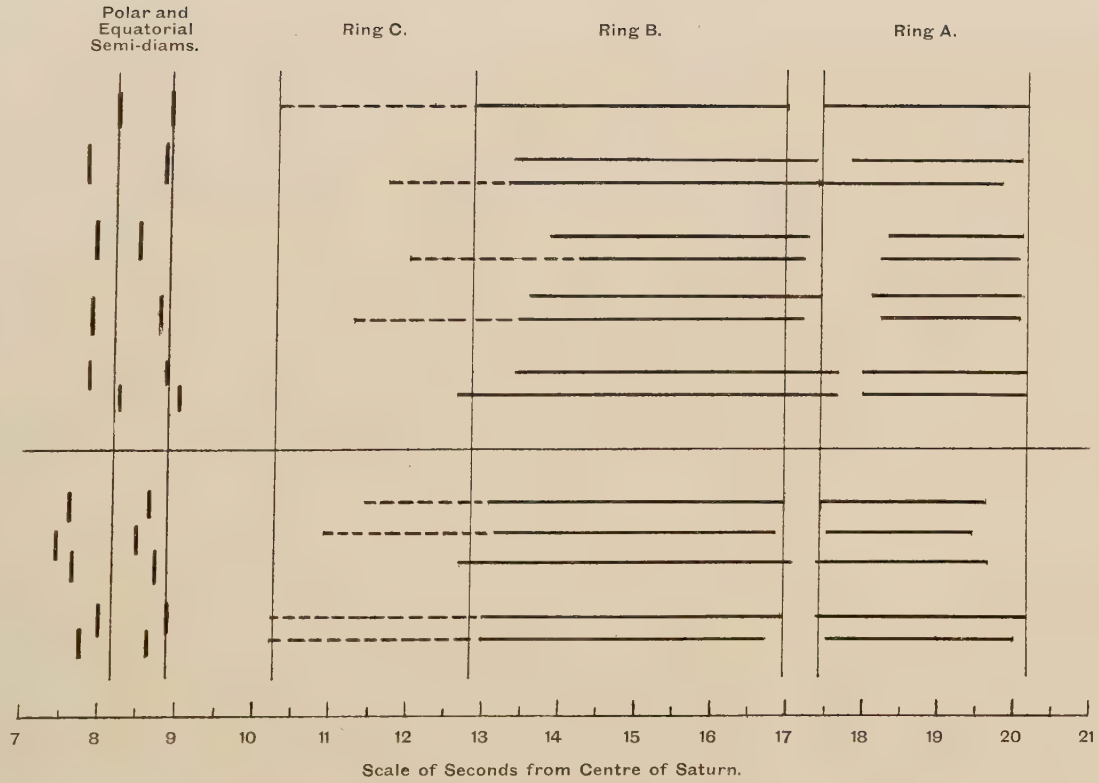
Polar and Equatorial Semi-diams.
Ring C.
Ring B.
Ring A.
1. Mean of Visual Measures.
2. Ho. (E.–W.)
3. Ho. (N.–S.)
4. He. (E.–W.) dense
5. He. (N.–S.) ,,
6. He. (E. W.) faint
7. He. (N.–S.) ,,
8. He. Image 10 dense
9. He. ,, faint
Measures from Photograph.
11. Nos. 4 and 5, reduced
12. Nos. 6 and 7 ,,
13. No. 9 ,,
14. See 1900
15. See corrected for Irradiation.
7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
Scale of Seconds from Centre of Saturn.

Table IV. compares such mean with the measures from the photograph of 1911. The same quantities are represented graphically in the upper part of the accompanying diagram. (Ho = Hollis. He = Hepburn.)

TABLE IV.

	1.	2.	3.	4.	5.	6.	7.	8.
		Measures from Photograph of 1911 Nov. 19.						
	Mean of Visual Measures 1 to 4.	Ho (East and West).	Ho (North and South).	He (East and West) (dense edges).	He (East and West) (faint edges).	He (North and South) (dense edges).	He (North and South) (faint edges).	He Image 10 (faint edges).
	″	″	″	″	″	″	″	″
Ring A extr.	40·375	40·186	39·684	40·204	40·204	40·120	40·120	40·66
,, intr.	34·902	35·716	...	36·617	36·200	...	36·448	36·00
Centre C. Div.	34·434	35·234	34·808	35·579	35·526	...	35·444	35·66
Ring B extr.	33·965	34·752	...	34·540	34·851	...	34·440	35·32
,, intr.	25·739	26·732	26·588	27·714	27·140	28·454	26·888	26·24
Ring C intr.	20·562	...	23·454	...	...	24·036	22·556	...
Saturn, equat. diameter	17·769	17·632	...	16·974	17·535	...	...	18·02
Saturn, polar diameter	16·348	...	...	...	...	15·83	15·78	...
Width, ring A	2·736	2·235	...	1·794	2·002	...	1·836	2·33
,, C. Div.	0·469	0·482	...	1·039	·675	...	1·004	0·34
,, ring B	4·113	4·010	...	3·413	3·855	...	3·776	4·54
,, ring C	2·576	...	1·567	...	...	2·209	2·166	...

Note.—Though expressed in seconds, the scale of columns 2 to 8 depends on the assumption stated above.

This table indicates several very marked discrepancies in the proportions of the system as measured on the photograph of 1911, compared with the proportions given by the visual measures of the earlier period. (These will hereafter be referred to as "measures (P)" and "measures (V)" respectively.) The following salient points appear, all of which are independent of the correctness of the scale adopted for the measures of the photograph.

1. Rings A and B are each proportionately narrower in (P) than in (V).

2. The ratio of the combined span of rings A and B to the space between the inner edge of ring B and the planet is much less in (P) than in (V).

3. Ring C extends about half-way across the space between ring B and the planet (P), while in (V) it extends about two-thirds of the way.

4. The above are independent of the scale adopted for the photographic measures, but such scale seems to be justified by the fact that the dimensions of the globe come out approximately equal in the two cases.

It is difficult, without supposing extensive real changes in the dimensions of the system, to conceive any hypothesis which will reconcile all the four points stated above. Three possible lines

of explanation suggest themselves: (1) a systematic error in the micrometric measurement of convex outlines as contrasted with concave outlines; (2) a difference in the visual and photographic images; (3) physical changes in the system.

As to (1), if there were a tendency to set the micrometer wire too far inward when measuring the concave limbs, the result would be to narrow the internal diameters of rings B and C, and so to produce an effect in the observed direction. In the case of the inner edge of ring A, such a tendency would probably be corrected by the near proximity of the convex edge of ring B. But this explanation leaves the increased ratio of the width of ring A to that of ring B in measures (V) unaccounted for.

Suggestion (2) practically amounts to this, that there is an excess in the effect produced in the visual image by irradiation over that produced in the photographic image by halation. To test this possible solution, the measures (P) have been reduced to the scale furnished by the Cassini division (since, on this hypothesis, some point within the boundaries of the Cassini division as determined by the measures (V) must coincide with some point within the same boundaries as determined by measures (P)). Let us take the point in the division at a distance of 17″·30 from the centre of the system* as determined by (V), and reduce the photographic measures to the scale given thereby. The result for the 5th and 8th columns of Table IV. is given in the 2nd and 4th columns of Table V., column 1 being the mean of measures (V) for comparison. (The figures of Table IV. are halved for an obvious reason.) The residuals obtained by subtracting the items of columns 2 and 4 respectively from column 1, changing the sign for inner edges, will then represent the corrections necessary to harmonise the visual and photographic measures. See's irradiation values are given in the 6th column for comparison.

TABLE V.

(*Semi-diameters.*)

	1. Mean Measures (V).	2. Mean Measures (P) (faint edges).	3. Res. (1)−(2).	4. Measures (P), Image 10.	5. Res. (1)−(4).	6. See's Irradiation.
	″	″	″	″	″	″
Ring A extr.	20·19	19·45	·74	19·65	·54	·15
,, intr.	17·45	17·52	·07	17·40	−·05	·15
Ring B extr.	16·98	16·86	·12	17·07	−·09	·25
,, intr.	12·87	13·13	·26	12·68	−·19	...†
Ring C extr.	10·28	10·89	·61	...	...	...
Saturn, equat. diameter	8·89	8·48	·41	8·71	·18	·28

* This point is one-third of the width of the division within its outer edge. According to See, irradiation at the inner edge is nearly double that at the outer edge.

† It is not very clear why no allowance should be made for irradiation at the inner edge of ring B. In the following discussion it is assumed that irradiation at this edge is ″·20.

Considering columns 2 and 3, we find that (for the edges of the Cassini division) the correction to bring the reduced measures (P) up to the visual dimensions is almost exactly one-half of See's correction for irradiation, implying that in this case halation affects the photographic image about half as much as irradiation affects the visual. In the case of the over-exposed image 10, the minus residuals indicate that halation exceeds irradiation, and to get a comparison with the visual figures we must apply about $-\frac{2}{5}$ of See's correction. Applying these corrections to all the dimensions (changing the sign for inner edges as before), we get the following table, in which any substantial residuals represent discrepancies which irradiation (or halation) seems inadequate to explain.

TABLE VI.

(*Semi-diameters.*)

	1. Mean Measures (V).	2. Mean Measures (P) (faint edges).	3. Res. (1)−(2).	4. Measures (P), Image 10.	5. Res. (1)−(4).
	″	″	″	″	″
Ring A extr.	20·19	19·52	·67	19·59	·60
,, intr.	17·45	17·45	·00	17·46	·01
Ring B extr.	16·98	16·98	·00	16·97	·01
,, intr.	12·87	13·03	·16	12·76	−·11
Equat. diam.	8·89	8·62	·27	8·60	·29
Width, ring A	2·74	2·07	·65	2·13	·61
,, ring B	4·11	3·95	·06	4·21	−·10

In this form there is a close correspondence between the dimensions obtained from the normally exposed and the over-exposed images, and, so far as the dimensions of ring B are concerned, we get a good approximation to the visual measures (see lower part of diagram). The discrepancies for the inner edge of ring B may be attributed to the fact that the transition from this ring into ring C is indefinite. No conclusions can be safely drawn as to the inner edge of ring C, owing to the impossibility of being sure that we see the actual edge projected on the bright globe of the planet. The outstanding features of this table are the marked deficiencies in the outer edge (and therefore in the width) of ring A, and in the equatorial radius. The results for the two sets of images are consistent with each other, and, taking their mean, we find a difference from the visual mean of no less than 1″·27 for the external diameter of ring A, and ″·55 for the equatorial diameter. Irradiation seems quite unable to account for either of these differences. In the case of ring A, it may be suggested that the outer zone is too faint to impress itself on the photographic plate, but this seems negatived by the facts that the results for the over-exposed and normally exposed images are consistent, and that the edges of the former image are perfectly sharp and well defined. In the case of the equatorial diameter an element of uncertainty is introduced by the great falling off of the light at the limb, the

extreme limb seen visually being almost black. But this again is perfectly sharp on the fully exposed image 10, and one would have been disposed to expect that any difference between the visual and photographic images arising from this cause would have been in the reverse direction to that observed.

Results similar in form to those in Tables V. and VI. are obtained from consideration of Mr. Hollis's measures, and the measures of the dense edges.

As to (3) If any reliance can be placed on the deficiency for the equatorial radius in Table VI., it indicates that the reduction of the scale of the measures (P) to that of the Cassini division, as shown by the visual measures, is excessive, implying a general shift outwards of all the members of the ring system.

It is noteworthy that such a shift is consistent with a set of measures made on eight nights during the apparition of 1912–13 by the Rev. T. E. R. Phillips, which, by his kind permission, are here summarised in

	″
Ring A (exterior)	40·037
Centre C. D.	34·755
Ring B (interior)	26·582
Equat. diameter	17·788

As in the measures of the photograph, the Cassini division and the inner edge of ring B are here given considerably greater diameters than those given by the older visual measures.

The large deficiency for ring A may indicate a real fluctuation in the apparent dimensions of this ring. Having regard to abnormalities of this ring appearing on the photograph, namely, its transparency, the non-concentricity of its outer edge, and its local differences of brightness about to be referred to, such a fluctuation does not seem inherently improbable.

The following matters of interest appear on examination of the photograph. They are observable consistently on all the images except two or three.

1. Transparency of ring A. This is clearly observable on all the images except No. 10. The combined brightness of ring and ball where the one is superposed on the other is greater than that of either separately (near the region in question), notwithstanding the fact that, except for a narrow fringe to the south, the ring is projected on to its own shadow. No sign of this shadow is visible. Some remarks by M. Trouvelot on the possible transparency of this ring are stated at the end of this paper.

2. The brightness of ring A is not uniform. The ring is divided longitudinally into two zones by a fairly well-marked boundary, which is, however, not concentric with either edge of the ring, but approaches the inner edge in the north-preceding and south-following quadrants, and the outer edge in those opposite. The inner zone is the brighter, and in the ansæ is the narrower, in the proportion of about 4 to 5. Perhaps as a consequence of the

broadening of the bright zone in the south-preceding and north-following quadrants, these are considerably brighter than their alternates. The Cassini division is better defined, and to a greater distance from the ansæ in the faint than in the bright quadrants.

3. The inner edge of ring B shows an apparent extension inwards where it crosses the ball, and shades off into the dark zone of ring C more gradually than would be expected from its sharpness in the ansæ. It is curious that the combined effect of ring A and ball is one of increased brightness, while that of ring C and ball is one of darkness.

Appendix.

In a memoir entitled "Sur la Variabilité des Anneaux de Saturne," published in the *Bulletin Astronomique* of Nov. 1884 and Jan. 1885, M. Trouvelot lays great stress on certain observations made by him in November and December 1880, when he noted that the shadow of ring A on the ball was not black but grey. On certain occasions, only the part of the shadow closely bordering on the edge of the ring was black, the outer zone being grey. He discusses these observations in detail, and cites similar observations by Bond, Webb, Dawes, and Huggins. In some of the observations cited, no shadow was seen, when it ought to have been visible. He also refers to the hypotheses suggested by Dawes and Bond that these appearances might be explained by atmospheres surrounding Saturn and the rings, or that the Sun's light may reach the planet through the ring. Having dismissed the first hypothesis, he proceeds: "Pour ma part, je ne vois guère que la transparence des anneaux A et C qui puisse en rendre compte. Mais, si l'anneau A est transparent, il est évident qu'il ne l'est pas toujours, et il est même supposable qu'il ne l'est que rarement, puisque, en général, l'ombre qu'il porte sur Saturne est noire comme encre, et que sa bordure est nettement arrêtée."

He then discusses the possibility of seeing the limb of Saturn through the ring, as in the case of ring C, but concludes that there is no chance of so observing it, owing to the underlying shadow: "L'occultation d'étoiles brillantes par cet anneau serait plutôt de nature à indiquer sa transparence."

Errata in Mr. W. S. Franks' Measures of Double Stars.

Page 519,	OΣΣ 19.	*For* 99°·95 *read* 98°·95.
„ 521,	„ 92.	„ 58″·55 „ 58″·05.
„ 523,	„ 131.	„ 212°·65 „ 212°·55.
„ 528,	ΣI 18.	*Insert* + sign before the Decl.
„ 529,	„ 41.	*For* 23 Sagittæ *read* 2, 3 Sagittæ.
„ 530,	Σ 735.	„ 30°·50 „ 35°·5.
„ 532,		Σ is *omitted* in the headings.
„ 534,	H*h* 781.	After τ^2 Aquarii *add* 294°·95 : 132″·36 : 2*n*.